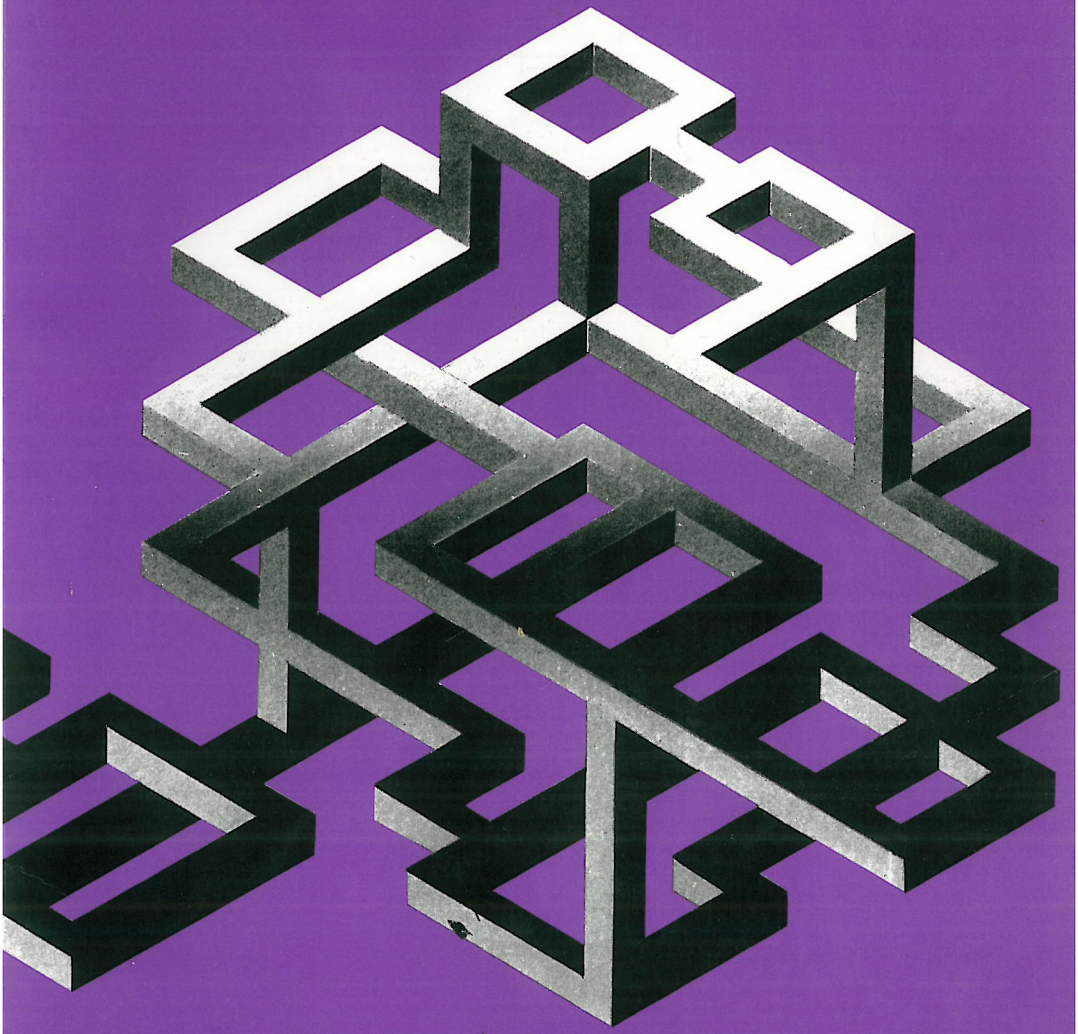
ST(P) Mathematics 4A

Teacher's Notes and Answers



L Bostock
S Chandler

Second Edition

A Shepherd
E Smith

ST(P) MATHEMATICS 4A

Teacher's Notes and Answers

L. Bostock, B.Sc.

S. Chandler, B.Sc.

A. Shepherd, B.Sc.

E. Smith, M.Sc.

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INTRODUCTION

This book completes coverage of Levels 8 and 9 of the national curriculum. Book 5A covers Level 10 and consolidates the course.

Multiple choice questions are included in this book. Even if practice on these is not required for examination purposes, they are valuable teaching aids. Such questions often make pupils think about problems in a different way and, if done in class, often provoke useful discussions. Multiple choice questions are also an effective way to force reluctant pupils into thinking about the reasonableness, or otherwise, of an answer, especially if they are not given sufficient time to do much calculation.

Those questions that are double underlined, e.g. $\underline{\underline{2.}}$, should be used cautiously if at all, with the less able. They are intended to give the brightest pupils food for thought but can easily damage the confidence of others. Questions which are single underlined, $\underline{2.}$, are extra, but not harder, questions. They can be used as extra practice, for faster workers or later for revision.

The text, though adequate, is brief and leaves ample scope for teachers to use their own methods and ideas, and to supplement the examples given. For a pupil who is revising a topic, the explanatory text is a useful reminder of the reasons for the methods followed.

Calculators should now be used fairly fully. In most cases, the required degree of accuracy is stated. When a result is required correct to three significant figures, then any intermediate working should be written down to four or five (no more) significant figures. In general, angles should be given correct to one decimal place and lengths calculated correct to three significant figures.

When pupils use calculators there is a strong tendency for them always to give answers correct to three significant figures or to give all the figures in the display, regardless of context. Pupils should be encouraged to think about the degree of accuracy appropriate to a given situation. They should also be encouraged to use appropriate units. For example they should realise the absurdity of giving the height of a tree to the nearest centimetre and the inappropriateness of giving the distance between Coventry and Birmingham in metres. There are questions in this book where the degree of accuracy required is not given and questions where the units required are not stated. These can be used for discussion.

The detailed notes that follow are only suggestions. Experienced teachers will have their own ideas on approach and order of content.

Exercise 1a (page 1)

Revise factorisation of quadratics before this revision exercise.

1. $\frac{7x + 10}{12}$
2. $\frac{14x - 17}{20}$
3. $\frac{13x - 4}{10}$
4. $\frac{x + 4}{42}$
5. $\frac{6x + 11}{12}$
6. $\frac{7x + 17}{10}$
7. $\frac{7x - 2}{12}$
8. $\frac{19x - 1}{10}$
9. $\frac{x + 14}{12}$
10. $\frac{7x + 2}{30}$
11. $\frac{7x + 2}{6}$
12. $\frac{2x + 13}{12}$
13. $\frac{13}{4x}$
14. $\frac{3}{65a}$
15. $\frac{5x + 17}{(x + 3)(x + 4)}$
16. $\frac{2(x + 10)}{(x - 4)(x + 3)}$
17. $\frac{11}{2(x + 2)}$
18. $\frac{13}{3x}$
19. $\frac{1}{12a}$
20. $\frac{7x + 1}{(x + 3)(x - 1)}$
21. $\frac{5x - 53}{(x + 7)(x - 4)}$

Exercise 1b (page 2)

1. $\frac{3x - 1}{(x + 1)(x - 1)}$
2. $\frac{3x - 1}{(x + 2)(x - 2)}$
3. $\frac{-(4x + 13)}{(x + 4)(x - 4)}$
4. $\frac{4x + 11}{(x + 3)(x - 3)}$
5. $\frac{x - 9}{(x + 2)(x - 2)}$
6. $\frac{2x + 9}{(x + 1)(x - 1)}$
7. $\frac{3x - 17}{(x + 5)(x - 5)}$
8. $\frac{-(9x + 58)}{(x + 7)(x - 7)}$
9. $\frac{4x - 13}{(x + 4)(x - 4)}$
10. $\frac{3x - 8}{2x(x - 2)}$
11. $\frac{5x - 9}{3x(x + 3)}$
12. $\frac{5x - 12}{(x + 3)(x - 3)}$

Exercise 1c
(page 3)

1. $\frac{1}{x-1}$
2. $\frac{1}{2-x}$
3. $\frac{1}{x-4}$
4. $\frac{1}{x+1}$
5. $\frac{2}{x+1}$
6. $\frac{1}{2x+1}$
7. $\frac{-1}{x+3}$
8. $\frac{1}{(x+1)(x+2)}$
9. $\frac{1}{x-3}$
10. $\frac{1}{x-4}$
11. $\frac{1}{x-2}$
12. $\frac{2}{(x+1)(x-3)}$
13. $\frac{-3}{x+2}$
14. $\frac{-4}{2x+1}$
15. $\frac{2}{x+4}$
16. $\frac{3}{(x+2)(x+5)}$

Exercise 1d
(page 4)

1. $\frac{15x+11}{12}$
2. $\frac{5x-2}{20}$
3. $\frac{1}{6x}$
4. $\frac{2(3x+4)}{(x+2)(x-2)}$
5. $\frac{-(2x+13)}{(x-1)(x+2)(x-4)}$
6. $\frac{2}{x-2}$
7. $\frac{2}{(x-2)(x-4)}$
8. $\frac{1}{(2x-1)(3x+1)}$

Exercise 1e
(page 5)

1. 6
2. 8
3. 5
4. 12
5. 24
6. 20
7. 20
8. 14
9. 2
10. 4
11. 4
12. $\frac{1}{2}$
13. 2
14. 3
15. 3
16. $\frac{1}{3}$
17. 2
18. 3
19. 3
20. 2
21. -2
22. 2
23. 2
24. 4
25. 3

Exercise 1f (page 7)

1. 2, 3
2. −5, 4
3. −1, 5
4. −3, 4
5. $-\frac{7}{2}$, 3
6. 4, 10
7. −1, −3
8. −3, 4
9. $\frac{1}{2}$, 3
10. $-5\frac{1}{6}$, 4
11. $-2\frac{1}{2}$, 5
12. 4, 20
13. $-1\frac{1}{2}$, $-1\frac{1}{4}$
14. 1, 2
15. $-4\frac{1}{3}$, 2
16. 8, 1

CHAPTER 2 Congruent Triangles

Exercise 2a (page 9)

1. $\hat{BAC} = 38.7°$, $\hat{ABC} = 51.3°$, $\hat{ACB} = 90°$; Yes
2. No
3. $\hat{BAC} = 106°$, $\hat{ABC} = 39.9°$, $\hat{ACB} = 34.1°$; Yes
4. No
5. The length of one side

Exercise 2b (page 10)

1. Yes; SSS
2. No
3. No
4. No
5. No
6. Yes; SSS
7. Yes; SSS
8. △ABC and △ADC; SSS
9. Yes; SSS
10. Yes; SSS or SAS

Exercise 2c (page 14)

5. Two are: △ABC and △LMN
6. Two

Exercise 2d (page 14)

1. Yes; AAS
2. No; similar
3. Yes; AAS
4. Yes; AAS
5. No
6. Yes; AAS
7. Yes; AAS
8. No
9. Yes
10. Yes

Exercise 2e (page 17)

1. Yes; AC = 4.4 cm, $\hat{A} = 34.3°$, $\hat{C} = 115.7°$
2. No
3. Yes; PR = 7.2 cm, $\hat{R} = 46°$, $\hat{P} = 74°$
4. No
5. Yes; DF = 7.8 cm, D = 50°, F = 40°

Exercise 2f (page 18)

After this exercise has been completed it is sensible to discuss when knowing two sides and a non-included angle of a triangle gives a unique triangle and when it doesn't.

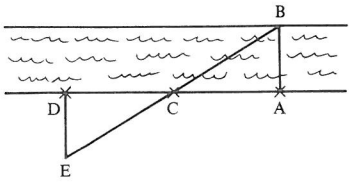
1. Yes
2. No; there are two possibilities
3. Yes
4. No; there are two possibilities
5. Yes
6. Yes
7. In questions 1, 3 and 5 we can calculate the length of the third side.

Exercise 2g (page 19)

Here is a practical application of congruent triangles:

This is a way of finding the width of a river without having to cross it. Put a stake A in the ground on one bank opposite a bush or tree B on the other bank. Walk along the bank at right angles to AB for a reasonable distance and put in another stake C. Carry on walking in the same straight line until you have covered a distance equal to AC. Put in another stake D. Now walk at right angles to AD until you are in a straight line with BC. Place another stake E. DE is the width of the river.

Draw a diagram illustrating this and explain why it works.
This basic idea, but using similar triangles, is repeated in Exercise 8b, question 10.



1. Yes, SAS
2. Not necessarily
3. Yes; SHR
4. Yes; SAS
5. Yes; SAS
6. Not necessarily
7. Yes; SHR
8. Not necessarily
9. Yes; SHR
10. Yes

Exercise 2h (page 22)

1. No
2. Yes; AAS
3. Yes; SSS
4. Yes; AAS
5. No
6. Yes; SAS
7. No; similar
8. Yes; ASS
9. Yes; SHR
10. Yes; SSS
11. Yes; ASS
12. Yes; SAS
13. No
14. No; similar
15. Yes; SHR

Exercise 2i (page 25)

9. $\triangle BDF$ and $\triangle CDE$

Exercise 2j (page 28)

Questions 5 to 10 are demanding for all except the most able pupils.

Exercise 2k (page 34)

This provides useful extra practice on congruent triangles but can be omitted.

1. AC bisects both angles; Yes
2. Both are right angles
3. No
4. They are equal
5. Yes; No; Yes, of AC; they are all right angles
6. No
7. They are equal

Exercise 2l (page 36)

If Exercise 2k was not covered, a reminder of the properties of special quadrilaterals is needed before this exercise is attempted. It includes several numerical and constructional questions. Remind pupils that 'construct' means 'make an accurate drawing of'.

1. 5 cm
2. 5 cm
3. 60°
5. 8 cm
6. 5 cm
7. 9.5 cm
8. 6 cm
9. 5.7 cm

CHAPTER 3 Prisms and Pyramids

Exercise 3a (page 40)

Revises earlier work but with harder examples. Calculators should be used and pupils encouraged to check results by estimation.

1. 35 700 cm^3
2. 13.7 cm^3
3. 130 m^3
4. 432 cm^3
5. 1680 mm^3

6. a) 1 000 000 or 10^6 b) 4 230 000 cm^3
7. a) 1000 b) 0.628 cm^3

8. 4.2 litres
9. 48 000 cm^3
10. 75 m^3
11. 0.432 m^3
12. 7.8 cm^3
13. 42 000 mm^3

14. 13.3 cm
15. 2.29 m
16. 0.625 cm
17. 2.11 mm
18. 3530 cm^3
19. 20 cm

20. a) 120 000 cm^3 or 0.12 m^3 b) 50 cm or 0.5 m
21. 864 cm^3

Exercise 3b (page 43) Revises earlier work but with harder examples.

1. 2175 cm^3
2. 32 000 cm^3
3. 1920 cm^3
4. 17.6 m^3
5. 55.44 m^3
6. 0.66 cm^3

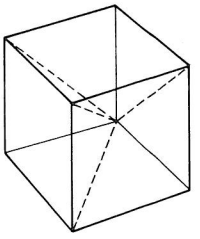
7. 88 cm^3
8. 2580 cm^3
9. 0.72 m^3
10. 1728 cm^3
11. 9 cm
12. 3.2 m^2

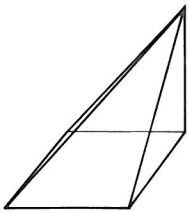
13. a) 44 cm^2 b) 9 cm
14. a) 12 cm b) 60 cm^2 c) 20 cm
15. 26 cm
16. a) 0.05 cm b) 0.5 mm
17. 14.4 cm
18. 144 cm^3
19. a) 47.1 cm^3 b) 2830 cm^3
20. a) 13 cm b) 60 cm^2 c) 1200 cm^3
21. a) 2250 m^3 b) 0.03 m^3 c) 5 hrs 13 mins

VOLUME OF A PYRAMID

Nets for making solids to demonstrate that the volume of a pyramid is $\frac{1}{3}$ area of base × perpendicular height.

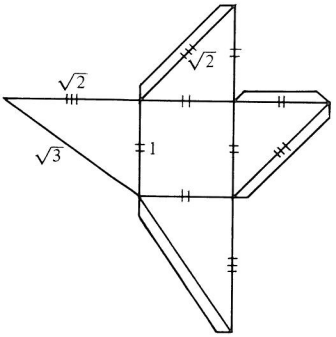
First Method





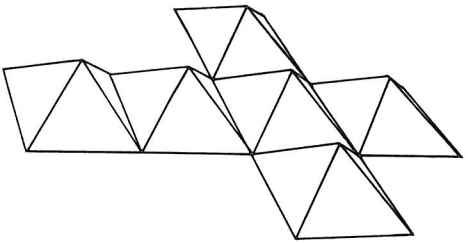
A cube can be formed from three identical pyramids each with a square base.

Net

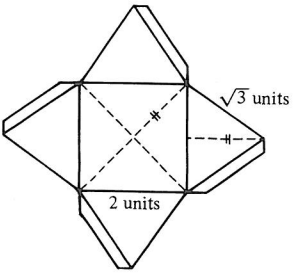


Start by drawing the square, then draw the two smaller triangles, finishing with the two larger triangles. Make sure the indicated lengths are equal.

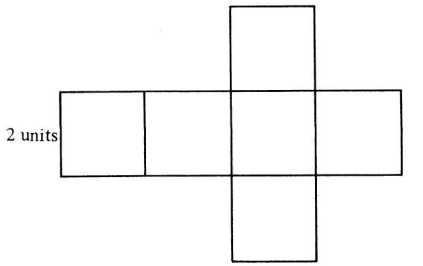
Second Method



This will fold up into a cube with the vertices of the six pyramids at the centre.



Make six pyramids and stick their bases to the six squares below.



Exercise 3c (page 49)

1. 72 cm^3 **2.** 640 cm^3 **3.** 80 cm^3 **4.** 960 m^3 **5.** 118 cm^3 **6.** 38.4 cm^3

7. 226 cm^3
8. a) 10 cm, 5 cm b) 12 cm c) 192 cm^3
9. a) 7.50 cm b) 165 cm^3

Exercise 3d (page 53)

1. 71.4 g **2.** 106 g **3.** a) 19 800 g, 19.8 kg **4.** 980 g **5.** 618 g **6.** 13.8 g

7. 17.2 g **8.** 0.69 g **9.** 8.9 g **10.** 0.55 g **11.** 2.5 g per cm^3 **12.** 2.6 g per cm^3 **13.** 62 cm^3 **14.** 2450 cm^3 **15.** 1990 g

16. a) 2.83 cm b) 90.5 cm^3
c) i) 1450 g ii) 1.45 kg d) £10 100 (to nearest £100)
17. a) 0.112 cm^3 b) 0.392 g

Exercise 3e (page 55)

1. B **2.** D **3.** B **4.** C

CHAPTER 4 Formulae

The answers given are not the only possible version. For instance $z = x + \frac{y}{100}$ may be given as $z = \frac{100x + y}{100}$ and $p = 2l + 2b$ may be given as $p = 2(l + b)$

Exercise 4a (page 56)

1. a) 95 p b) $C = 25x + 15y$

2. a) 40 ° b) $y = 180 - 2x$

3. $n = \frac{a+b}{2}$
4. $p = 2l + 2b$
5. $C = \frac{pm}{100}$
6. $C = A + nD$
7. $p = 10q$
8. $T = n + 1$
9. $t = 2n - 1$
10. $b = 3c + 10$
11. $z = x + \frac{y}{100}$
12. $s = 3n + 3$

Exercise 4b (page 57)

Revise directed numbers.

1. a) 15 b) 14.4
2. a) –1 b) 2.575
3. a) $\frac{5}{8}$ b) 12
4. a) 77 b) 21.2
5. a) 63 b) 7.56
6. 452 cm^2
7. 3.59 s
8. 330 cm^2
9. 11 300 m or 11.3 km
10. 12.3 J
11. 23.1 m

Exercise 4c (page 59)

1. a) 6 b) 3
2. 9
3. a) $\frac{3}{4}$ b) 1.8
4. a) 17 b) $2\frac{1}{3}$
5. a) 2 b) 1.6
6. a) 6 b) 0.25
7. $1\frac{5}{7}$
8. 22
9. a) 10 cm b) 20 cm
10. 12 sides
11. a) 150 miles b) 52 people

Exercise 4d (page 61)

1. 2
2. –5
3. $9 - a$
4. $\frac{q}{6}$
5. $q - p$
6. $\frac{n}{m}$
7. $e + f$
8. $\frac{g}{h}$
9. $g + h$
10. $k - h$
11. $r = p - q$
12. $s = r + t$
13. $t = r - s$
14. $z = \frac{y}{x}$
15. $m = n - l$
16. $Q = \frac{P}{R}$
17. $a = 2s - b - c$
18. $r = \frac{C}{2\pi}$
19. $b = \frac{A}{l}$
20. $u = v - at$

Exercise 4e (page 62)

1. 2 **4.** $1\frac{1}{2}$ **7.** $\frac{b+c}{a}$ **9.** $\frac{c-ab}{a}$

2. 4 **5.** $\frac{r-q}{P}$ **8.** $\frac{a-c}{b}$ **10.** $\frac{2+pq}{P}$

3. 2 **6.** $\frac{c-d}{b}$

11. $d = ab - c$ **16.** $l = 2 + mn$

12. $a = \frac{c+d}{b}$ **17.** $m = \frac{l-2}{n}$

13. $q = \frac{1-pr}{p}$ **18.** $P = \frac{4T-Q}{2}$

14. $P = \frac{2+3Q}{3}$ **19.** $p = \frac{m-mr}{r}$ or $p = \frac{m(1-r)}{r}$

15. $t = \frac{s-u}{7}$ **20.** $y = \frac{2+zx}{x}$

21. a) 45. 2 b) 11 c) $a = \frac{P-4b}{4}$ d) 11, yes

22. a) 1017.6 b) –2 c) $a = \frac{A-3nl}{3n}$ d) −2, yes

23. a) 2.1 b) $\frac{1}{2}$ c) $y = \frac{x-z}{z}$ d) $\frac{1}{2}$, yes

Exercise 4f (page 64)

In the worked example, as there is a choice, the answer may be of the form $x = \frac{d-c}{b-a}$

1. $2\frac{1}{2}$ **5.** $\frac{c}{a-b}$ **9.** $\frac{a-c}{b+d}$

2. $\frac{3}{5}$ **6.** $\frac{b}{a-c}$ **10.** $\frac{a-c}{b-d}$ or $\frac{c-a}{d-b}$

3. $\frac{2}{5}$ **7.** $\frac{2q}{p-r}$ **11.** $\frac{p}{q+r+s}$

4. 2 **8.** $\frac{s}{p+t}$ **12.** $\frac{a+b+d}{c}$

Questions 11 to 14 need careful attention with many examples.

13. $\frac{c}{a+1}$ **15.** $\frac{d}{c-1}$ **17.** $p = \frac{r}{q+s}$ **19.** $a = \frac{c}{b-d}$

14. $\frac{4}{b-1}$ **16.** $\frac{a}{1-2a}$ **18.** $a = \frac{d}{b+c}$ **20.** $a = \frac{c-b}{x-y}$

21. $p = \dfrac{-qr}{q+r}$

22. $a = \dfrac{b}{c-1}$

23. $q = \dfrac{pr-p}{r+1} = \dfrac{p(r-1)}{r+1}$

24. $r = \dfrac{p+q}{p-q}$

25. 5

26. $\frac{2}{3}$

27. 10

28. $\dfrac{b}{a-b}$

29. $\dfrac{ab+c}{a}$

30. $\dfrac{ab}{b-a}$

31. $\dfrac{ac+bc}{b-a} = \dfrac{c(a+b)}{b-a}$

32. $\dfrac{cd-ab}{a-c}$

33. $q = \dfrac{pr}{1-p}$

34. $a = \dfrac{bc}{b-c}$

35. $s = \dfrac{ut}{2t+u}$

36. $n = \dfrac{ml}{1+m}$

37. $p = \dfrac{q}{q+r}$

38. $R = \dfrac{PQ}{P-Q}$

39. $b = \dfrac{ac}{a-c}$

40. $u = \dfrac{2st}{t-s}$

Exercise 4g (page 66)

The difference between type (a) and type (b) in the worked example needs to be made clear.

1. 8

2. 5

3. $\frac{5}{2} = 2\frac{1}{2}$

4. $\frac{2}{3}$

5. $4\frac{1}{2}$

6. $2\frac{1}{2}$

7. $\frac{1}{8}$

8. 100

9. 1000

10. ± 12

11. 4

12. ± 11

13. ± 30

14. ± 10

15. -3

Exercise 4h (page 67)

1. ± 2

2. $\pm\frac{5}{3}$

3. $\pm\sqrt{\frac{5}{3}}$

4. $\pm\sqrt{p}$

5. $\pm\sqrt{\dfrac{q}{p}}$

6. $\pm\dfrac{q}{\sqrt{p}}$

7. $\pm\sqrt{p+q}$

8. $\pm\sqrt{\dfrac{bc}{a}}$

9. 16

10. $\frac{4}{9}$

11. 27

12. a^2

13. $\dfrac{q^2}{p^2}$

14. $\dfrac{r^2}{p}$

15. p^2q

16. $16-a$

17. $p = \pm\frac{\sqrt{q}}{2}$

18. $p = \frac{a^2}{4}$

19. $a = b^2 - x$

20. $a = \pm\sqrt{c-b}$

21. $A = C^2 - B$

22. $h = \frac{2D^2}{3}$

23. $b = z - a$

24. $x = \pm\sqrt{b^2 - a^2}$

25. a) ± 3 b) ± 5

26. 3 seconds

27. a) 4 b) 20
c) $Q = P^2 - R$ d) 20, yes

Exercise 4i (page 69)

It is most important that fractions are removed as soon as possible otherwise the solution is either unnecessarily complicated or, as often as not, wrong.

1. 24

2. $7\frac{1}{2}$

3. $1\frac{3}{5}$

4. $2\frac{2}{5}$

5. pq

6. $\frac{pq}{q-p}$

7. $\frac{pr}{a}$

8. $r(p+q)$

9. $a(c-b)$

10. $\frac{a^2+b^2}{b-a}$

11. a

12. $\frac{bc}{a+b}$

13. $R = \frac{100I}{PT}$

14. $n = \frac{2A}{a+l}$

15. $Q = 4P - R$

16. $b = \frac{4a+3c}{6}$

17. $x = \frac{ab}{a+b}$

18. $x = p+q+r$

19. $x = \frac{s-r}{t}$

20. $q = \frac{4p}{a^2}$

21. $l = \frac{gT^2}{4\pi^2}$

22. $H = \frac{ht}{2h-t}$

23. $X = \frac{-(b^2+c)}{ba^2}$

24. $B = \frac{2aM+bL}{L}$

Exercise 4j (page 70)

1. $t = \frac{v-u}{a}$

2. $h = \frac{2A}{b}$

3. $c = \pm\sqrt{a^2-b^2}$

4. $h = \frac{2A}{a+b}$

5. $f = \frac{uv}{u+v}$

6. $a = \frac{2A-bh}{h}$

7. $v = \dfrac{2s - ut}{t}$

8. $t = \dfrac{2s}{u + v}$

9. $u = \dfrac{vf}{v - f}$

10. $a = \dfrac{v^2 - u^2}{2s}$

11. $h = \dfrac{A - \pi r^2}{\pi r}$

12. $u = \pm\sqrt{v^2 - 2as}$

13. $a = \pm\dfrac{\sqrt{v^2 + \omega^2 x^2}}{\omega}$

14. $h = \pm\dfrac{\sqrt{A^2 - \pi^2 r^4}}{\pi r}$

15. $u = \dfrac{2s - at^2}{2t}$

16. $a = \dfrac{2s - 2ut}{t^2}$

17. $p = \dfrac{2A}{q \sin R}$

18. $u = \pm\sqrt{\dfrac{mv^2 - 2E}{m}}$

19. $g = \dfrac{4\pi^2 l}{T^2}$

20. $R = \dfrac{100A - 100P}{PT}$

Exercise 4k (page 71)

1. $6\frac{4}{5}$

2. 7

3. $c = \dfrac{a^2 - b^2 d}{b^2}$

4. $T = \dfrac{100I}{PR}$

5. 7

Exercise 4l (page 71)

1. -0.55

2. ± 6

3. $d = T(v - u)$

4. $p = \dfrac{q^2}{16}$

5. $b = \dfrac{ac}{c - a}$

Exercise 4m (page 72)

1. C 2. C 3. A 4. C 5. D

CHAPTER 5 Graphs

A list of useful techniques for drawing curves was given in a previous book. These points are important and should be repeated. They are:

1. Do not take too few points. About ten are usually necessary.
2. To decide where to draw the x-axis, look at the range of y-values.
3. To decide where to draw the y-axis, look at the range of x-values.
4. In some questions most of the y-values are given but some have to be calculated. In this case always plot first those points that were given and from these, get an idea of the shape of the curve. Then plot the points that were calculated and see if they fit onto the curve you have in mind. If they do not, go back and check the calculations.

5. To draw a smooth curve to pass through the points, always turn the page into a position where the wrist is on the inside of the curve.

Some pupils can investigate the graph of $x = y^2$. Show that all we have done is interchange the x and y axes.

Accurate graph drawing is time-consuming but the use of a graph-drawing package on a computer can extend the number of curves that can be investigated and show that very accurate solutions to equations can be obtained.

Exercise 5a (page 75)

1. a) 3.25, 1.5 b) i) – 0.30, 3.30 ii) 1, 2
2. a) –1, 2 b) i) 0.27, 3.73 ii) 2 iii) – 0.24, 4.24
3. a) 4.65, – 0.65 b) 5.79, –12.64
4. a) 2, 3 b) – 0.25 c) 0.4, 4.6
5. a) 5, $x = 1$ b) 2.1 c) – 0.4, 2.4
6. a) $6\frac{1}{4}$, $x = -\frac{3}{2}$ b) 0.6, –3.6

Exercise 5b (page 76)

1. a) –1, 4 b) –1.7, 4.7 c) –1.5, 4.5 d) 0.4, 2.6
2. a) – 0.85, 4.85 b) 0.25, 3.7 c) – 0.65, 4.6 d) – 0.24, 4.24
e) 0.59, 3.41
3. a) 1, 3 b) 3.7, 0.3 c) 4.2, – 0.2
4. a) 3.3, – 0.3 b) 3.6, – 0.6
No, the line $y = 5$ does not intersect the graph
5. a) 5.4, 0.6; $x^2 - 6x + 3 = 0$
b) 4.4, 1.6
6. a) –1, 2 b) –1.8, 2.79
7. a) 1.9 b) 0.72, 2.78
8. a) –3.91, 0.9 b) –3.31, 0.3, $x^2 + 3x - 1 = 0$
9. a) No solutions b) –1.38, 1.7 c) – 0.44, 0.77
10. –3.25, 1.24
11. –5.54, 0.54

Exercise 5c (page 83)

1. a) $x^2 - x - 7 = 0$ b) $x^2 - 2x - 5 = 0$
c) $x^2 - 6x + 4 = 0$ d) $x^2 + 3x - 5 = 0$
2. a) $y = 2x + 1$ b) $y = 7x - 2$
c) $y = -6x - 4$ d) $y = -\frac{7}{2}x - 1$
3. –1.56, 2.56; $x^2 - x - 4 = 0$
4. –2.62, 7.62; $x^2 - 5x - 20 = 0$
5. ± 3.46; $x^2 + 2x - 5 = 0$; –3.45, 1.45

6. From -3.74 to 1.07; $3x^2 + 8x - 12 = 0$
7. From -1.74 to 5.74; $x^2 - 4x - 10 = 0$
8. -0.39, 3.89; from 0.44 to 4.56; 0.44, 4.56; $8x^2 - 33x - 16 = 0$
9. $y = \frac{1}{2}x + 6$; 2.71, -2.21
10. $y = 2 - 5x$; -5.37 and 0.37

These are calculated values. This accuracy is not attainable from a sketch.

11. 15, 0, 3; 0.68, 3.32; $4x^2 - 16x + 9 = 0$; from 0.40 to 3.10; 0.40, 3.10; $4x^2 - 14x + 5 = 0$

Exercise 5d (page 89)

1. a) $-2 < x < 2$ b) $x < -5, x > 5$ c) $-1 \leqslant x \leqslant 1$ d) $x \leqslant -6, x \geqslant 6$
2. a) $0 < x < 3$ b) $x \leqslant 0, x \geqslant 4$ c) $-2 < x < 0$ d) $-1 \leqslant x \leqslant 0$
3. a) $(x - 2)(x - 1)$ c) $x^2 = 3x - 2$ d) $1 \leqslant x \leqslant 2$
4. $2 \leqslant x \leqslant 4$

Exercise 5e (page 90)

1. $0 < x < 6$
2. $-5 < x < 0$
3. $-7 < x < 7$
4. $x < 1, x > 4$
5. $x < 0, x > 9$
6. $-1 < x < 7$

Exercise 5f (page 91)

It is worth pointing out that the local maximum or minimum does not necessarily occur at one of the points given in the table.

1. a) 2.71 b) -2.47
2. a) 3.68 b) -3.42
3. a) -3.1 b) 3.1
4. -3
5. 1.75
6. a) $y = x$ b) -1, 0, 1

Exercise 5g (page 95)

Discuss the problem that arises as the value of x gets close to zero. Hence justify the range of values of x given in each question.

1. One a) 0.77 b) -0.63
2. a) 2.61 b) $x > 2.14$ c) Lowest value is 1 when $x = 12$
3. a) 1.5, 10.5 b) $x^2 - 12x + 16 = 0$ c) From 1.5 to 10.5
4. a) -4
   b) $x^2 + 2x - 8 = 0$; two; draw the graph of $y = \frac{8}{x}$ for values of x from 1 to 8
5. a) y gets smaller and smaller b) No c) No
6. From 0.65 to 4.60, $4x^2 - 21x + 12 = 0$
7. a) $y = x + 1$ b) No. There is a negative solution

Exercise 5h (page 87)

1. D **2.** B **3.** A **4.** C **5.** B **6.** A **7.** C

By the end of this chapter pupils should have a good idea of the shape of a parabola, cubic curve and hyperbola and be able to recognise the forms of equation that give rise to these curves. Encourage shape recognition by asking them to sketch, without axes, the curves of a variety of these equations.

CHAPTER 6 Indices

Exercise 6a (page 100)

Revises earlier work but with more algebraic examples

1. 81
2. 16
3. 144
4. 64
5. 144
6. $\frac{49}{25}$
7. $\frac{1}{2}$
8. 6
9. 4
10. 1
11. 1
12. $1\frac{1}{3}$
13. 4
14. 27
15. 32
16. 27
17. 25
18. $\frac{9}{16}$
19. $6\frac{1}{4}$
20. $\frac{8}{27}$

21. 1
22. $\frac{27}{125}$
23. $\frac{5}{2}$
24. 64
25. b^5
26. c^2
27. c^2
28. $\frac{1}{x^4}$
29. b^7
30. $\frac{b}{a}$
31. b^4
32. y^2
33. x^2
34. $\frac{4}{d^2}$
35. $\frac{1}{x^3}$
36. $\frac{1}{p^2}$
37. 81
38. 64
39. 15 625
40. x^8

41. a^{10}
42. x^6
43. 512
44. 729
45. 256
46. x^{15}
47. y^8
48. x^{-6}
49. $16a^5$
50. $4p$
51. $\frac{2}{x^2}$
52. $18x^3$
53. $2x$
54. $\frac{1}{2a}$
55. $4x^4$
56. $15y^3$
57. $\frac{4}{a^2}$
58. $12x^5$
59. $24y^4$
60. $\frac{5}{y^2}$

Exercise 6b (page 103)

When revising standard form remind pupils about scientific notation on calculators and how to 'read' the display.

1. 2.8×10^2
2. 3.9×10^{-1}
3. 7.07×10^2
4. 9.7×10^{-2}
5. 2.77×10^3
6. 8×10^{-5}
7. 8×10^{-1}
8. 8×10^3
9. 2.5×10^{-2}
10. 8.4×10^5
11. 1.08×10^{10}
12. 1.54×10^{-4}
13. 1.15×10^{-5}
14. 3.2×10^2
15. 7.8×10^{-2}
16. 2×10^3
17. 7×10^4
18. 3×10^{-2}
19. 1.4×10^{-5}
20. 3×10^0
21. 1.25×10^8
22. 3.2×10^3
23. 3.2×10^{-2}
24. 3.31×10^5
25. 4.13×10^{-3}
26. 2.59×10^{-2}
27. 2.8×10^6

28. a) 6×10^3 b) 2.4×10^6 c) 1.2005×10^5
29. a) 8.64×10^{-12} b) 6×10^{-2} c) 1.128×10^{-5}
30. a) 1.3×10^3 b) 5.2×10^7 c) 2.6005×10^3 d) 2.5995×10^4

Exercise 6c (page 106)

Emphasise, repeatedly, that $\sqrt{4}$, $4^{\frac{1}{2}}$, . . . mean the positive root. If the negative root is required we write $-\sqrt{4}$, $-4^{\frac{1}{2}}$ and if both are required, $\pm\sqrt{4}$, $\pm 4^{\frac{1}{2}}$.

1. 3
2. 4
3. 6
4. 2
5. 5
6. 4
7. $\frac{1}{2}$
8. 0.2
9. $\frac{1}{2}$
10. $\frac{2}{3}$
11. 0.5
12. $\frac{2}{3}$

Exercise 6d (page 107)

1. 9
2. $\frac{1}{4}$
3. 8
4. 25
5. 0.04
6. 1728
7. 0.216
8. 27
9. 4
10. 100
11. 0.001
12. 100
13. 3
14. $3\frac{1}{2}$
15. 5
16. $1\frac{1}{2}$
17. $\frac{1}{4}$
18. $\frac{1}{8}$
19. $\frac{1}{2}$
20. 1000
21. 0.01
22. 27
23. $11\frac{1}{9}$
24. 0.4
25. $x^{\frac{1}{2}}$
26. x^2
27. y^2
28. a^2
29. x^6
30. x^4

Exercise 6e (page 108)

1. 2.88
2. 4.90
3. 3.16
4. 2.93
5. 0.215
6. 1.48
7. 7.95
8. 2.45
9. 0.381
10. 1.08
11. 0.681
12. 2.22

Exercise 6f (page 108)

1. C 2. B 3. D 4. B 5. D 6. C 7. A

Exercise 6g (page 109)

1. a) 4 b) $\frac{1}{7}$ c) 16 d) 125
2. a) $\frac{1}{4}$ b) 3 c) 1 d) $\frac{27}{16}$
3. a) $\frac{1}{16}$ b) 1 c) $\frac{2}{5}$ d) $12\frac{1}{2}$
4. a) 8.1×10^6 b) 2.73×10^4 c) 9×10
5. a) $\frac{1}{x}$ b) $\frac{2}{a^2}$ c) $\frac{y}{x^2}$
6. a) 2 b) 4

Exercise 6h (page 109)

1. a) $\frac{9}{16}$ b) $1\frac{2}{3}$ c) 2 d) 1
2. a) 4.32×10^{-5} b) 7.5×10^{-2} c) 2.58×10^{-2} d) 2.22×10^{-2}
3. a) 5 b) $\frac{1}{36}$ c) $\frac{1}{125}$ d) $\frac{6}{25}$
4. a) $2\frac{1}{4}$ b) $\frac{9}{25}$ c) $2\frac{1}{2}$ d) $1\frac{19}{81}$
5. a) p^2 b) $\frac{1}{2x}$ c) x^{12}
6. a) 3 b) 3

Exercise 6i (page 110)

1. a) $\frac{16}{81}$ b) $\frac{2}{3}$ c) $5\frac{4}{9}$ d) 8
2. a) 64 b) 25 c) 4 d) 8
3. a) x^2 b) $\frac{3}{2y^3}$ c) y^8
4. a) 6.4×10^9 b) 1.6×10 c) 6.25×10^{-2} d) 3×10^5
5. a) p^{10} b) $x^{\frac{3}{2}}$ c) $\frac{2}{5y}$
6. a) $\pm\frac{2}{3}$ b) $\frac{1}{3}$

CHAPTER 7 Cylinders, Cones and Spheres

The Value of π

The approximate value of π given by a scientific calculator is used in this chapter. If by any chance this value is not available then 3.142 may be used. Some examination papers specify the approximate value to be used so some practice in using 3.142 or $\frac{22}{7}$ may be needed.

Encourage estimation of answers: for this purpose use $\pi \approx 3$.

Exercise 7a (page 111)

1. 56.5 cm, 254 cm^2
2. 20.1 cm, 32.2 cm^2
3. 151 m, 1810 m^2
4. 146 mm, 1660 mm^2
5. 81.7 cm, 531 cm^2
6. 12.6 cm, 12.6 cm^2
7. 6.66 cm, 3.53 cm^2
8. 89.2 mm, 633 mm^2
9. 18.2 cm, 26.4 cm^2
10. 45.9 cm, 167 cm^2
11. 5.65 m, 2.54 m^2
12. 120 mm, 1150 mm^2
13. 55.3 cm, 243 cm^2
14. 647 mm, 333 cm^2
15. 7.54 m, 4.52 m^2
16. 119 cm, 1130 cm^2
17. 256 cm, 5200 cm^2
18. 584 mm, 272 cm^2
19. 6π cm, 9π cm^2
20. 24π cm, 144π cm^2
21. 160π cm, 6400π cm^2
22. 2π m, π m^2
23. 9π m, 20.25π m^2
24. 22π mm, 121π mm^2

25. 207 cm
26. 1520 cm^2
27. 49.7 cm^2
28. 92.5 cm, 509 cm^2
29. 201 cm^2

30. a) 244 m b) 213 m c) 1140 m^2
31. 56π cm^2 **32.** D **33.** C

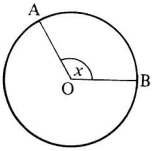
Exercise 7b (page 114)

1. 2.90 cm
2. 10.8 cm
3. 2.23 cm
4. 0.414 cm
5. 4.77 m
6. 30.6 m
7. 0.668 m
8. 17.2 cm
9. 2.07 mm
10. 23.6 mm
11. 15.8 cm
12. 7 cm
13. 0.54 m
14. 10.0 mm
15. 38 mm
16. 37.6 mm
17. 1.10 m
18. 45 cm
19. 3.66 cm
20. 6.15 m
21. 2.88 mm
22. 1.13 m
23. 6.28 cm
24. 4.22 cm
25. 1.69 cm
26. 19.5 mm
27. 2.03 m

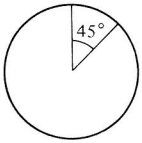
28. 35.0 mm, 3850 mm^2
29. 15.9 cm, 796 cm^2
30. 3.12 cm
31. 4.44 cm, 27.9 cm^2
32. 3.09 cm

Exercise 7c (page 117)

Some examples for discussion, are needed:
e.g.



If arc AB is $\frac{1}{3}$ of the circumference, what is the size of angle x?



What fraction of the area of the circle is the area of the sector?

1. 10.5 cm, 52.4 cm^2 **3.** 1.88 cm, 2.26 cm^2 **5.** 188 m, 6790 m^2
2. 10.5 cm, 26.2 cm^2 **4.** 4.19 cm, 6.70 cm^2 **6.** 3.14 m, 18.8 m^2

7. 85.7 cm, 78.5 cm^2 **8.** 85.7 cm, 370 cm^2 **9.** 12.8 cm, 9.24 cm^2

10. 66.8 ° **12.** 15.3 ° **14.** 8.91 cm
11. 43.0 ° **13.** 8.59 cm **15.** 3.44 m

16. a) 36 ° b) 7.86 cm^2
17. a) 12.6 cm c) 120 ° d) 37.7 cm^2 e) 37.7 cm^2
18. a) 8 cm^2 b) 4.57 cm^2
19. a) 495 m^2 b) 106 m^2

Exercise 7d (page 121)

1. 44 cm, 154 cm^2 **3.** 110 cm, $962\frac{1}{2}$ cm^2 **5.** 4.4 cm, 1.54 cm^2
2. 11 cm, $9\frac{5}{8}$ cm^2 **4.** 440 cm, 15 400 cm^2 **6.** $14\frac{2}{3}$ cm, $18\frac{2}{9}$ cm^2

7. 18 cm **8.** $38\frac{1}{2}$ cm^2 **9.** 22 m **10.** 154 cm^2

11. 7 cm **13.** $1\frac{3}{4}$ cm **15.** $1\frac{2}{5}$ m
12. 28 cm **14.** $5\frac{1}{4}$ cm **16.** 70 cm

Exercise 7e (page 123)

1. 151 cm^2 **5.** 1210 cm^2 or 0.121 m^2
2. 377 cm^2 **6.** 255 cm^2
3. 226 cm^2 **7.** 13 700 cm^2
4. 103 000 cm^2 or 10.3 m^2 **8.** 259 m^2

9. a) 377 cm^2 b) 113 cm^2 c) 603 cm^2 **11.** 209 cm^2
10. a) 96.5 cm^2 b) 161 cm^2 **12.** 928 cm^2

13. 4.40 m^2

Exercise 7f (page 124)

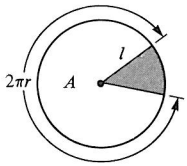
1. 430 cm^3
2. 257 cm^3
3. 21.2 m^3
4. 1020 mm^3
5. 1320 cm^3
6. 34 500 cm^3
7. 74 000 cm^3 or 0.074 m^3
8. 3.08 m^3
9. a) 61 100 cm^3 b) 61.1 litres
10. a) 43.2 cm^3 b) 834 g
11. a) 10 050 cm^2 b) 20 holes
12. 2.38 cm
13. 2.69 cm
14. 1.73 cm
15. 2.11 m
16. 16.8 cm
17. 1.22 cm
18. 1.27 m
19. a) 1 m^3 b) 1.03 m
20. 20 cm
21. 54.0 cm
22. 42 400 cm^3

Exercise 7g (page 128)

1. 1700 cm^3
2. 29.4 cm^3
3. 78 200 cm^3
4. 0.528 cm^3
5. 27.2 cm^3
6. 0.107 m^3
7. 1150 cm^3
8. 330 cm^3
9. 228 cm^3
10. 113 cm^3

Exercise 7h (page 130)

The most able pupils may be interested in the derivation of the formula $A = \pi rl$.



Considering the cone made from a sector of a circle then:

$$\frac{A}{\pi l^2} = \frac{2\pi r}{2\pi l} \Rightarrow A = \pi rl$$

1. 126 cm^2
2. 434 cm^2
3. 4.15 m^2
4. 15 200 mm^2
5. 163 cm^2
6. a) 302 cm^3 b) 10 cm c) 188 cm^2

Exercise 7i (page 131)

Tell the pupils that the formulae for the volume and curved surface area of a sphere cannot be proved at this stage.

1. 113 cm^3
2. 1560 cm^3
3. 230 000 cm^3
4. 0.998 cm^3
5. 24.4 m^3
6. 9200 mm^3
7. 262 cm^3
8. a) 145 cm^3 b) 4.52 cm
9. $\frac{9}{2}\pi$ cm^3

Exercise 7j (page 131)

1. 1020 cm^2
2. 254 cm^2
3. 21 100 cm^2
4. 10.2 cm^2
5. 3320 cm^2
6. (146 m^2) 6 pots

Exercise 7k (page 132)

1. 596
2. 572 cm^3
3. a) 15 cm b) 3020 cm^3
4. 239 000 cm^3
5. Sphere; 25.7 cm^3
6. a) $\frac{64}{3}\pi$ cm^3 b) 64π cm^3 c) $\frac{128}{3}\pi$ cm^3
7. a) $\frac{32}{3}\pi$ cm^3 b) $\frac{2048}{3}\pi$ cm^3 c) 64

Exercise 7l (page 133)

1. Sphere; 16.5 cm^2
2. 462 cm^2
3. a) 15 cm b) 679 cm^2
4. Total surface area = 32.4 m^2
   No
   Enough to cover 2.4 m^2 is still needed
5. 511 cm^2

Exercise 7m (page 135)

1. C
2. B
3. A
4. D
5. D
6. A
7. B
8. D
9. C
10. B

CHAPTER 8 Similar Shapes

Exercise 8a (page 138)

Revises earlier work on similar triangles.

1. Yes. The angles of the one triangle are equal to the angles of the other triangle
2. Yes. The three pairs of corresponding sides are in the same ratio
3. Yes. There is one pair of equal angles and the sides containing these equal angles are in the same ratio
4. Yes. $L\hat{M}N = Q\hat{P}R = 90°$ and the sides containing these angles are in the same ratio
5. Yes. The angles of $\triangle RST$ are equal to the angles of $\triangle ABC$
6. Yes. The three pairs of corresponding sides are in the same ratio
7. Yes. $B\hat{A}C = Q\hat{R}P = 85°$ and the sides containing these angles are in the same ratio
8. Yes. The angles of $\triangle ABC$ are equal to the angles of $\triangle DEF$
9. a) Each triangle contains a right angle and the sides containing the right angles are in the same ratio
   b) The value of each is $\frac{1}{2}$

10. Yes, $\frac{PR}{QR} = \frac{1}{2}$, $PR = 2\frac{1}{2}$ cm

11. $\frac{1}{2}$, 6 mm

12. The three angles of $\triangle ABC$ are equal to the three angles of $\triangle APQ$

13. Yes

Exercise 8b (page 143)

Nos 11 to 14 introduce the intercept theorem.

3. Yes, Yes **4.** 60° **6.** No **7.** 5 m **9.** 6 m

11. b) $QC = 3\frac{3}{4}$ cm c) $BQ = 2\frac{1}{4}$ cm d) 3 : 5

12. b) XL = 2.4 cm c) MZ = 6 cm d) 2 : 3

13. b) AD = 4 cm c) 4 : 1

14. b) AY = 10 cm c) 1 : 2

Exercise 8c (page 148)

1. 3 cm **3.** $2\frac{2}{3}$ cm **5.** 3 cm **7.** 7.5 cm **9.** 8 cm

2. $2\frac{2}{3}$ cm **4.** 6 cm **6.** 2.4 cm **8.** 1 cm **10.** 2 cm

Exercise 8d (page 151)

Inaccuracies frequently arise in drawing lines of a particular length. Pupils should be reminded that it is necessary to move the eye so that the eye, the mark on the ruler, and the mark on the paper are always in the same straight line. Stress the importance of a suitably sharpened pencil together with a correct handling of compasses and other instruments.

5. a) $2x$, $3x$, $4x$, $5x$
b) 1 : 4, 2 : 3, 3 : 2, 4 : 1

9. 7

Exercise 8e (page 153)

Questions 1 to 3 are suitable for class discussion.

1. a) 1 : 2 : 3 : 4
b) 1 : 4 : 9 : 16
Yes. Numbers in (b) are the squares of those in (a)

2. a) 1 : 2 : 3 : 4
b) 1 : 4 : 9 : 16
As for No.1

3. a) 1 : 2 : 3 : 4
b) 1 : 4 : 9 : 16
c) As for No.1

4. a) PR = 20 cm b) QS = 12 cm
c) $\triangle ABC = 30$ cm^2, $\triangle PQR = 120$ cm^2
d) 1 : 4

5. XW = 2 cm, $\triangle ABC = 9\ cm^2$, $\triangle XYZ = 4\ cm^2$

6. LN = 8 cm, $\triangle ABC = 25\ cm^2$, $\triangle LMN = 16\ cm^2$

7. BC = 9 cm, Area ABCD = $27\ cm^2$, Area PQRS = $3\ cm^2$

8. LP = 3 cm, Area WXYZ = $32\ cm^2$, Area LMNP = $18\ cm^2$

9.

Similar figures	Ratio of sides	Ratio areas
Triangles in question 5	3 : 2	9 : 4
Triangles in question 6	5 : 4	25 : 16
Rectangles in question 7	3 : 1	9 : 1
Parallelograms in question 8	4 : 3	16 : 9

Exercise 8f (page 156)

1. 4 : 1
2. 9 : 25
3. 4 : 9
4. 9 : 16
5. 25 : 9
6. 16 : 25
7. 1 : 1225
8. 1 : 400
9. 1 : 36
10. 5 : 3
11. 3 : 2
12. 2 : 1
13. 5 : 4
14. 7 : 4
15. 5 : 8
16. $8\ cm^2$
17. $16\ cm^2$
18. $7.5\ cm^2$
19. $64\ cm^2$
20. 6 : 5
21. 2.1 cm
22. $50\ cm^2$

Exercise 8g (page 163)

1. $1\frac{1}{2}\ cm^2$
2. 2 : 1
3. $200\ m^2$
4. 4 : 1
5. a) 2.25 cm b) $\frac{9}{49}$ c) $\frac{9}{40}$
6. 4 : 9, 2 : 3
7. a) i) 9 cm ii) 12 cm iii) $\frac{1}{16}$ iv) $\frac{9}{16}$
b) i) $16a$ ii) $3a$
8. AD = 2 cm

Exercise 8h (page 165)

A plentiful supply of cubes or cuboids would be most useful as an introduction to this exercise. Sets of similar containers, e.g. cylinders or jugs, may also help to demonstrate the point that is being made.

1. a) 1 : 2 : 3 b) 1 : 2 : 3 c) 1 : 8 : 27
2. a) i) 2 : 3 ii) 2 : 3 iii) 2 : 3
b) 8 : 27
3. a) 3 : 2 : 5 b) 27 : 8 : 125
4. a) i) 1 : 2 : 5 ii) 1 : 2 : 5
b) 1 : 8 : 125

Exercise 8i (page 168)

Much may be made of the practical nature of much of this exercise.

1. 8 : 1
2. 27 : 64
3. 2 : 3

4. a) 4 : 3 b) 4 : 3
5. a) 4 : 1 b) 8 : 1
6. a) 1 : 100 b) 1 : 1000 c) 1 : 10 d) 1 : 1

7. $1\frac{11}{16}$ pts, 4 pints

8. $4\frac{1}{2}$ p

9. 125 centilitres, 216 centilitres

10. a) 9 cm , 12 cm b) 64 cm^2, 100 cm^2
11. a) 1 : 50 b) 1 : 125 000 c) 3 cm d) 7500 cm^2 or 0.75 m^2
12. a) 12 : 13 b) 1728 : 2197
13. 64 kg
14. a) 21% b) 33%
15. a) 224% b) 483%
16. a) 44% b) 73%
17. a) 26% b) 59%
18. 36% 48·8%

Exercise 8j (page 171)

2. 7 : 5
3. b) i) QZ = 20 cm ii) XQ = 10 cm c) 1 : 2, 2 : 3
5. a) 3 cm b) 5 : 3 c) 3 : 5

Exercise 8k (page 172)

1. a) 7 : 9 b) 7 : 9
2. a) 45 cm^2 b) 4 : 9 c) 3 : 4
3. a) 3.2 cm b) 4 : 9 c) $\frac{16}{81}$
d) $\frac{16}{65}$ e) $\frac{4}{5}$ f) $\frac{4}{9}$
4. a) $\frac{1}{3}$ b) $\frac{1}{3}$ c) $\frac{2}{3}$
d) $\frac{2}{3}$ e) $\frac{1}{9}$ f) $\frac{4}{9}$
5. $\frac{1}{8}$

CHAPTER 9 Information Matrices

Exercise 9a (page 175)

1. a) $\begin{pmatrix} 100 & 300 & (50) \\ [200] & 0 & 300 \end{pmatrix}$
b)
c) 300

2. a) and b) $\begin{pmatrix} [50] & 25 & 37 \\ 100 & 150 & 89 \\ 92 & (250) & 340 \end{pmatrix}$
c) 339

3. a) and b) $\begin{pmatrix} 200 & 150 & 120 \\ 350 & 200 & \underline{70} \\ 190 & \textcircled{250} & 100 \\ 280 & 210 & 110 \end{pmatrix}$ 4×3

c) 400 d) 620

e) $\begin{pmatrix} 200 & 350 & 190 & 280 \\ 150 & 200 & 250 & \underline{210} \\ 120 & 70 & 100 & 110 \end{pmatrix}$

4. a) cost (p) potatoes carrots parsnips

$$\mathbf{M} = \begin{matrix} A \\ B \end{matrix} \begin{pmatrix} 10 & 8 & 12 \\ 12 & \textcircled{9} & 10 \end{pmatrix}$$

c) lb

$$\mathbf{P} = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} \begin{matrix} \text{potatoes} \\ \text{carrots} \\ \text{parsnips} \end{matrix}$$

d) 82 p

e) $\begin{pmatrix} 82 \\ 89 \end{pmatrix}$

f) Top entry gives the cost of Mr Smith's purchase in shop A
Bottom entry gives the cost of Mr Smith's purchase in shop B

5. a) 21 500
b) 15 000

c) $\mathbf{N} = \begin{matrix} A & B & C \\ (20 & 50 & 100) \end{matrix}$

d) The first and third entries in **NM** give the answers to (a) and (b)

6. a) 80

b) $\begin{pmatrix} 80 \\ 180 \end{pmatrix}$; gives the number of coins on each emptying

c) 1700 p

d) $\begin{pmatrix} 10 \\ 20 \\ 50 \end{pmatrix}$

e) The entries in **AV** give the amounts (in pence) of money

Exercise 9b
(page 178)

Question 8 is demanding. It involves route matrices. The most able will enjoy puzzling this one out themselves but use it for discussion with other pupils.

1. $\mathbf{NM} = (18 \quad 9 \quad 12)$ and lists the total number of matches won, drawn and lost

$\mathbf{MP} = \begin{pmatrix} 13 \\ 13 \\ 13 \end{pmatrix}$ and lists the number of matches played by each team

$\mathbf{M}\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 18 \\ 14 \\ 13 \end{pmatrix}$ and lists the points accumulated by each team

2. a)
$$\mathbf{TC} = \begin{matrix} \text{£} \\ \begin{pmatrix} 223 \\ 279 \\ 310 \end{pmatrix} \end{matrix}$$ and lists cost of raw materials each month

b)
$$\mathbf{TR} = \begin{matrix} \text{hrs} \\ \begin{pmatrix} 90 \\ 85 \\ 110 \end{pmatrix} \end{matrix}$$ and lists the time taken to make each month's orders

$$\mathbf{TC} + 10\mathbf{TR} = \begin{matrix} \text{£} \\ \begin{pmatrix} 1123 \\ 1129 \\ 1410 \end{pmatrix} \end{matrix}$$ and lists the total cost of each month's orders

3. a) $(7 \quad 10 \quad 39)$; lists the numbers of each type of milk ordered

b) $\begin{pmatrix} 21 \\ 20 \\ 15 \end{pmatrix}$; lists the number of bottles ordered by each customer

c) $\begin{pmatrix} 532 \\ 500 \\ 395 \end{pmatrix}$; cost (p) to each customer for the week

d) (1427); cost (p) of milk sold in the flats that week

4. a) $\begin{pmatrix} 17 \\ 16 \\ 32 \end{pmatrix}$; number of employees in each factory

b) $(33 \quad 18 \quad 14)$; number of employees in each category

c) $\begin{pmatrix}1580\\1420\\2700\end{pmatrix}$; the weekly wage bill for each factory

d) (5700); the total weekly wage bill for the three factories

5. a) $(107\ 000)$; total daily calorie requirements of all the people in the hostel

b) (85); the number of people in the hostel

6. a) $\begin{matrix}&\text{I}&\text{II}&\text{III}&\text{IV}\\ \text{A}\\ \text{B}\end{matrix}\begin{pmatrix}2100&2300&2300&1400\\1950&2340&2250&1560\end{pmatrix}$; cost of items ordered each quarter from source A and from source B

b) $\begin{pmatrix}8100\\8100\end{pmatrix}$

c) B
d) both the same

7. $\begin{pmatrix}128&106&34\\105&87&28\\174&144&46\end{pmatrix}$; the figures ringed show the week's pay of each employee

(261); the total week's pay of the three employees

8. $\begin{matrix}\text{C}&\text{D}&\text{E}&\text{F}\\ \begin{pmatrix}1&0&1&0\\1&1&0&1\\2&1&1&1\end{pmatrix}&\begin{matrix}\text{X}\\\text{Y}\\\text{Z}\end{matrix}\end{matrix}$

The entries show the number of train routes available between C, D, E, F and X, Y, Z

Exercise 9c (page 182)

Questions 4 and 5 involve route matrices and they are both *extremely* demanding. Pupils, other than the most able, should not try these without help.

1. $(5\quad 1\quad 1.5)$; $(5\quad 1\quad 1.5)\mathbf{R}\begin{pmatrix}1\\1\\1\end{pmatrix}$

2. $\mathbf{S}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$; $\begin{matrix}\\ \text{A}\\ \text{B}\end{matrix}\begin{pmatrix}40 & 30 & 25\\ 35 & 30 & 29\end{pmatrix}\mathbf{S}$

3. a) $\mathbf{F}\begin{pmatrix}1\\1\\1\end{pmatrix}$ b) $\mathbf{F}\begin{pmatrix}25\\28\\30\end{pmatrix}$ c) $(1\ 1\ 1)\ \mathbf{F}$

d) $(1\ \ 1\ \ 1)\ \mathbf{F}\begin{pmatrix}25\\28\\30\end{pmatrix}$

4. a) The top row shows the number of direct services available *from* X to X, Y and A. Similarly for the other rows
The first column shows the number of direct services *to* X from X, Y and A. Similarly the other columns

b) $\mathbf{R}^2 = \begin{matrix}\\ \text{X}\\ \text{Y}\\ \text{A}\end{matrix}\begin{matrix}\begin{matrix}\text{X} & \text{Y} & \text{A}\end{matrix}\\ \begin{pmatrix}9 & 1 & 0\\ 0 & 9 & 3\\ 3 & 0 & 0\end{pmatrix}\end{matrix}$

The entries in the main diagonal show the number of ways it is possible to travel from one place to another and then back again without a break in either journey

5. a) $\mathbf{FA} = \begin{pmatrix}1 & 1 & 0\\ 1 & 2 & 1\end{pmatrix}$ b) $\mathbf{BF} = \begin{pmatrix}1 & 0\\ 1 & 1\\ 2 & 1\end{pmatrix}$

c) $\mathbf{BFA} = \begin{matrix}\\ \text{X}\\ \text{Y}\\ \text{Z}\end{matrix}\begin{matrix}\begin{matrix}\text{L} & \text{M} & \text{N}\end{matrix}\\ \begin{pmatrix}1 & 1 & 0\\ 1 & 2 & 1\\ 2 & 3 & 1\end{pmatrix}\end{matrix}$

The numbers 2 and 3 indicate that there are 2 and 3 air routes respectively between the towns indicated by the positions of the entries

CHAPTER 10 Geometric Proof

This chapter attempts to show how to give a reasoned argument and why this is desirable. The examples in the exercises are not numerical, but reasoned solutions to numerical problems are expected at this level and should be insisted upon. Most pupils working at Levels 9–10 appreciate the need for a theoretical proof and many will be stimulated by a discussion

on Euclidean Geometry and its structure. This gives the pupils an insight into mathematics as an academic subject in its own right as opposed to its use as a tool for other subjects.

Pupils, other than the most able, may find the exercises, as given in this chapter, difficult. In this case change them to numerical questions; for example when a relationship between angles has to be proved, give a size for one angle and ask for the size of the other angle.

Exercise 10a (page 188)

It is worth developing number 6 to show how all the other circle angle facts can be deduced from this proof.

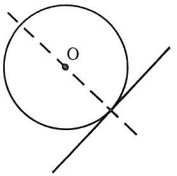
CHAPTER 11 Circles and Tangents

If Chapter 10 has not been covered, a revision of all the geometry contained in that chapter is desirable.

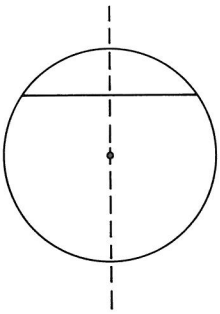
Exercise 11a (page 197)

1. A straight line parallel to the ground, 20 cm above it.
   a) one b) 20 cm c) radius, 90°

2.



3.



4. b) A tangent, touching the circle at N. 90°

Exercise 11b (page 200)

Use at least one question for discussion, emphasising the need for a rough sketch before embarking on the actual construction. Also remind the pupils of the need for a *sharp* pencil.

1. OB = 5 cm, CB = 2 cm
2. 30°
3. $x = 50°$, $y = 40°$
4. $x = 20°$, $y = 70°$

5. $x = 40°$, $y = 50°$
6. AB = 12 cm, $\hat{OBA} = 22.6°$
7. $x = 30°$, $y = 60°$, $z = 60°$
8. 30°
9. 5 cm
11. 9.80 cm (correct to 3 s.f.)

Exercise 11c (page 202)

2. $\hat{CAP} = 90°$. PA and PB are tangents to the circle centre C
4. a) 4 cm

Exercise 11d (page 203)

With the less able use only for discussion.

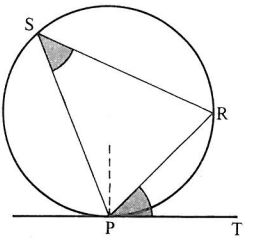
Exercise 11e (page 204)

1. $p = 65°$, $q = 65°$
2. $e = f = 67°$, $i = g = 23°$, $h = 134°$
3. a) 50° b) a kite (also a cyclic quadrilateral)
4. $x = 96°$, $y = 48°$
5. $r = 36°$, $s = 36°$
6. a) 6 cm b) 73.7° (to 1 d.p.)
7. 8 cm
8. Yes
9. 5.77 cm (to 3 s.f.)
10. 60°, 8 cm, 13.9 cm (to 3 s.f.)
11. 19.3 cm, 13.7 cm, 13.7 cm (to 3 s.f.)
13. They are equal

Alternate Segment Theorem

Demonstrations of this theorem should be given before the formal proof. If the demonstrations suggested in the answer book for 3A (page 39) were used, and the pieces of card are still available, it can be repeated. Also constructing and measuring is convincing:

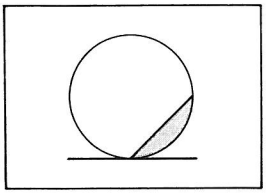
e.g.



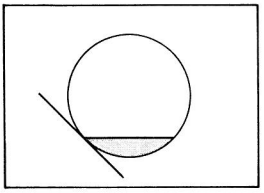
Construct a tangent to a circle (radius $\simeq$ 5 cm) and draw any chord, PR, then complete the figure using any point S on the circumference. Measure the shaded angles.

Exercise 11f
(page 209)

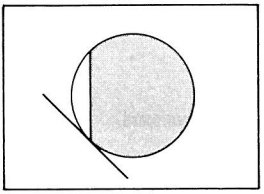
1.



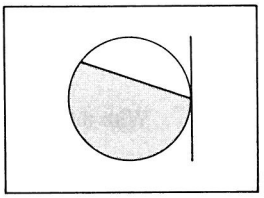
3.



2.



4.



5. $d = 73°$, $e = 26°$, $f = 81°$
6. $p = q = r = s = 60°$
7. $k = m = l = 64°$, $n = 52°$
8. $u = v = 67°$
9. $q = 57°$, $p = 57°$
10. $w = 90°$, $x = 27°$, $y = 117°$
11. $d = e = f = 47°$, $g = 86°$
12. $e = f = g = 54°$, $h = 72°$
13. $k = 74°$, $l = m = 53°$
14. $r = 90°$, $t = 35°$, $s = 55°$
15. $f = g = h = 71°$, $i = 38°$
16. $d = 90°$, $e = f = g = 45°$, $h = 90°$
17. $s = 30°$, $t = 60°$, $u = 60°$, $v = 10°$
18. $x = 28°$, $y = 62°$, $z = 62°$
19. $i = k = l = 37°$, $j = 53°$
20. $x = 60°$, $y = 61.5°$, $z = 58.5°$

CHAPTER 12 Probability

Exercise 12a
(page 214)

This exercise revises probability, as covered in 3A.

1. a) $\frac{1}{9}$ b) $\frac{2}{9}$ c) $\frac{2}{9}$

2. a) $\frac{1}{8}$ b) $\frac{3}{8}$ c) $\frac{1}{2}$

3. $\frac{2}{3}, \frac{1}{3}$ a) $\frac{4}{9}$ b) $\frac{4}{9}$ c) $\frac{4}{9}$

Exercise 12b (page 215)

1. a) i) $\frac{5}{8}$ ii) $\frac{3}{8}$ b) $\frac{3}{7}$ c) $\frac{4}{7}$

2. a) i) $\frac{1}{13}$ ii) $\frac{1}{4}$ b) $\frac{4}{17}$ c) $\frac{4}{51}$

3. a) $\frac{4}{9}$ b) i) $\frac{3}{8}$ ii) $\frac{5}{8}$ c) i) $\frac{1}{2}$ ii) $\frac{1}{2}$

Exercise 12c (page 216)

1. a) $\frac{4}{9}$ b) $\frac{1}{2}$ c) $\frac{1}{6}$ d) $\frac{5}{18}$

2. a) $\frac{2}{5}$ b) $\frac{1}{3}$ c) $\frac{1}{3}$

3. a) $\frac{3}{7}$ b) $\frac{1}{7}$ c) $\frac{2}{7}$ d) $\frac{2}{7}$ e) $\frac{2}{7}$ f) $\frac{4}{7}$

4. a) $\frac{3}{4}$ b) $\frac{9}{16}$

5. a) $\frac{2}{3}$ b) $\frac{5}{8}$ c) $\frac{1}{2}$ d) $\frac{5}{21}$

Exercise 12d (page 219)

1. a) $\frac{15}{28}$ b) $\frac{13}{28}$

2. $\frac{8}{15}$

3. $\frac{5}{12}$

4. a) $\frac{1}{8}$ b) $\frac{3}{8}$

5. a) $\frac{1}{16}$ b) $\frac{1}{16}$ c) $\frac{1}{16}$ d) $\frac{1}{16}$ e) $\frac{1}{4}$

6. $\frac{9}{17}$

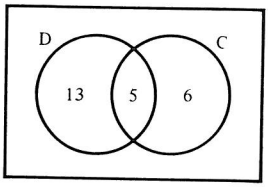
7. a) $\frac{1}{9}$ b) $\frac{7}{18}$

8. $\frac{7}{15}$

Exercise 12e (page 221)

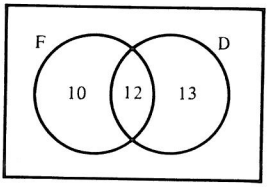
1. a) $\frac{1}{9}$ b) $\frac{4}{27}$ c) $\frac{13}{27}$

2.



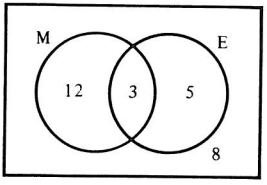
a) $\frac{11}{24}$ b) $\frac{13}{24}$ c) $\frac{19}{24}$

3.



$\frac{23}{35}$

4.



a) 28 b) $\frac{5}{7}$

5. a) $\frac{5}{12}$ b) $\frac{7}{12}$ c) $\frac{7}{36}$

6. a) $\frac{7}{16}$ b) $\frac{19}{32}$

7. a) 52 b) i) $\frac{19}{26}$ ii) $\frac{9}{52}$ iii) $\frac{21}{52}$

8. $x = 5$ a) $\frac{4}{21}$ b) $\frac{8}{21}$ c) $\frac{8}{21}$ d) $\frac{3}{7}$

Exercise 12f (page 224)

1. a) i) $\frac{27}{100}$ ii) $\frac{9}{100}$ iii) $\frac{1}{5}$ b) i) $\frac{9}{1000}$ ii) $\frac{43}{250}$

2. a) $\frac{1}{4}$ b) $\frac{1}{6}$ c) $\frac{1}{3}$

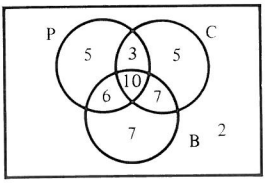
3. a) $\frac{1}{5}$ b) $\frac{24}{145}$ c) $\frac{506}{1015}$ d) $\frac{509}{1015}$

4. a) i) $\frac{1}{9}$ ii) $\frac{4}{45}$ b) i) $\frac{1}{18}$ ii) $\frac{7}{90}$

5. a) $\frac{4}{25}$ b) $\frac{1}{50}$ c) $\frac{1}{20}$ d) $\frac{8}{125}$ e) $\frac{1}{1000}$

6. a) $\frac{4}{9}$ b) Bob

7.



a) $\frac{1}{9}$ b) $\frac{26}{45}$ c) $\frac{17}{45}$ d) $\frac{7}{45}$

CHAPTER 13 Statistics

Exercise 13a (page 227)

Definitive answers to questions 1 to 3 are not possible. Some ideas are given here, but they may not be appropriate for the circumstances in any one school. Accept any reasonable responses provided that they are supported with valid arguments. These questions can be used for discussion and the topic can be taken further by considering samples used in magazines, etc.

1. a) Probably the older pupils, as younger pupils tend to arrive early.
b) There may be a slight bias in favour of girls as they tend to arrive earlier than boys.
c) No, because some older pupils do not have to arrive on time if they do not have a lesson.
d) Yes, because the later a pupil arrives at school, the more likely they are to be generally disorganised, particularly about breakfast.

2. a) Probably not, but it would be safer to select from each band.
b) Unlikely, as the total numbers in each year of the sixth form tend to be smaller than in younger year-groups.
c) This very much depends on how the intake to the school each year varies.

3. a) The first method will give a reasonably representative sample if there are equal numbers of boys and girls. One possible drawback is that selecting children whose surnames start with letters at and near the beginning of the alphabet may not be representative of the ethnic mix in the school. The second method is likely to give a representative sample.
b) NFER takes a 10% sample of pupils from a school by selecting pupils born on the 10th, 20th and 30th of each month.

4. In 1936, only the better-off sections of the population had telephones and this section of the population would also have been more inclined to buy magazines; i.e. a whole strata of the population was excluded from the sample.

5. This question investigates the reliability of sample sizes. The answers will obviously vary and we have given answers calculated from probability. The question can be extended to a discussion of the economics of sampling; e.g. small samples cost less than bigger ones but can involve a bigger cost when the sample gives the 'wrong' answer.
a) 1 f) 0.4 h) 0.7

Exercise 13b
(page 229)

1.

Weekday	Number of lunches served	Running total of lunches served
Monday	126	126
Tuesday	154	280
Wednesday	144	424
Thursday	175	599
Friday	118	717

2.

Place	Distance from Cardiff
Newport	10
Severn Bridge	26
Leigh Delamere	54
Swindon	72
Reading	111
London airport	139
Central London	154

3.

Day	Amount spent	Running total of expenditure
Monday	12	12
Tuesday	26	38
Wednesday	5	43
Thursday	8	51
Friday	32	83
Saturday	27	110
Sunday	4	114

Exercise 13c
(page 231)

1.

Score	Frequency	Score	Cumulative frequency
0	3	$\leqslant 0$	3
1	8	$\leqslant 1$	$3 + 8 = 11$
2	4	$\leqslant 2$	$11 + 4 = 15$
3	3	$\leqslant 3$	$15 + 3 = 18$
4	5	$\leqslant 4$	$18 + 5 = 23$
5	2	$\leqslant 5$	$23 + 2 = 25$
6	1	$\leqslant 6$	$25 + 1 = 26$

2.

Mark	Frequency	Mark	Cumulative frequency
1–10	7	$\leqslant 10$	7
11–20	14	$\leqslant 20$	21
21–30	18	$\leqslant 30$	39
31–40	33	$\leqslant 40$	72
41–50	36	$\leqslant 50$	108
51–60	43	$\leqslant 60$	151
61–70	21	$\leqslant 70$	172
71–80	15	$\leqslant 80$	187
81–90	8	$\leqslant 90$	195
91–100	5	$\leqslant 100$	200

a) 200 b) 108 c) 49

3.

Score	$\leqslant 19$	$\leqslant 39$	$\leqslant 59$	$\leqslant 79$	$\leqslant 99$	$\leqslant 119$	$\leqslant 139$
Cumulative frequency	8	22	55	61	66	69	70

a) 70 b) 55 c) 48

4.

Score	67	68	69	70	71	72	73	74	75	76	77	78
Frequency	2	4	9	9	12	15	13	8	5	8	6	4

a) 13 b) 23

5.

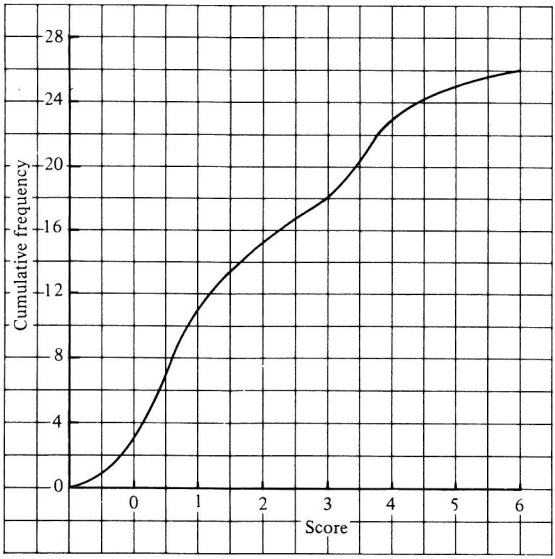
Number of books sold	0–5	6–10	11–15	16–20	21–25
Frequency	77	124	182	228	164
Number of books sold	$\leqslant 5$	$\leqslant 10$	$\leqslant 15$	$\leqslant 20$	$\leqslant 25$
Cumulative frequency	77	201	383	611	775

Number of books sold	26–30	31–35	36–40	41–45	46–50
Frequency	92	73	32	22	9
Number of books sold	$\leqslant 30$	$\leqslant 35$	$\leqslant 40$	$\leqslant 45$	$\leqslant 50$
Cumulative frequency	867	940	972	994	1003

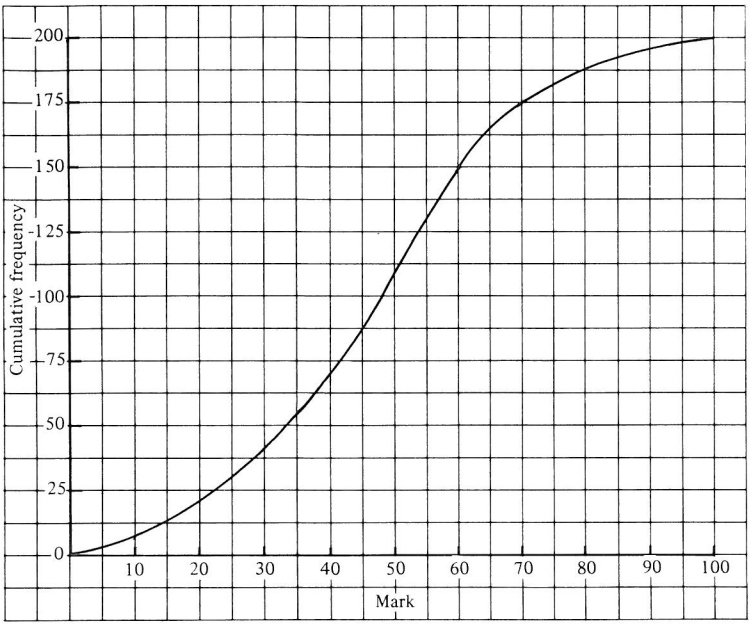
a) 136 b) 611 c) 666
It could have been shared

Exercise 13d
(page 234)

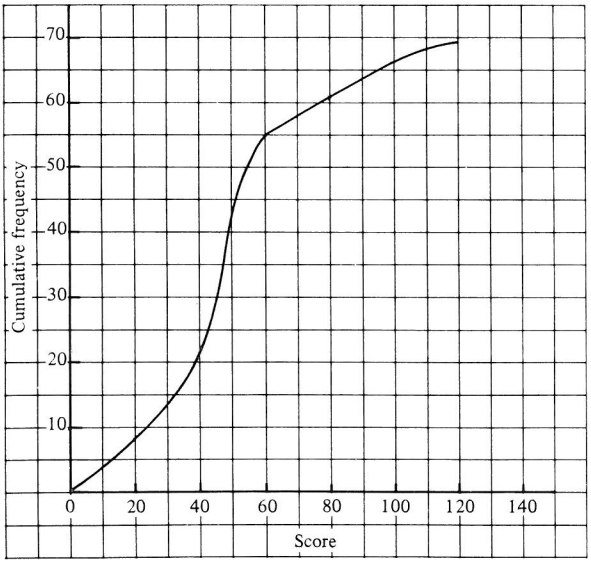
1.



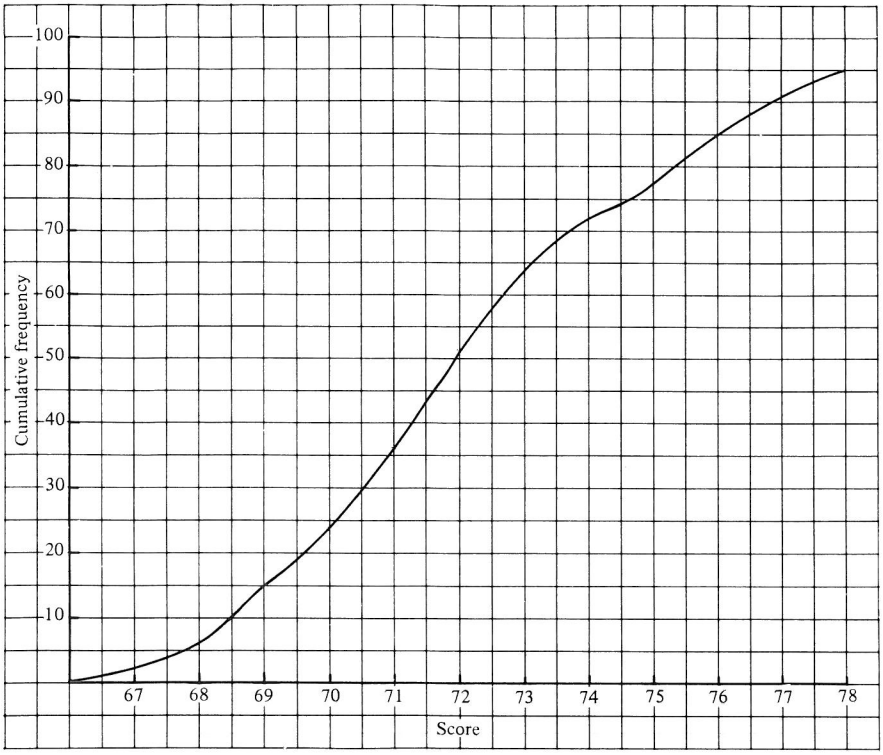
2.


200
175
150
125
100
75
50
25
0
Cumulative frequency
10
20
30
40
50
60
70
80
90
100
Mark

3.


70
60
50
40
30
20
10
Cumulative frequency
0
20
40
60
80
100
120
140
Score

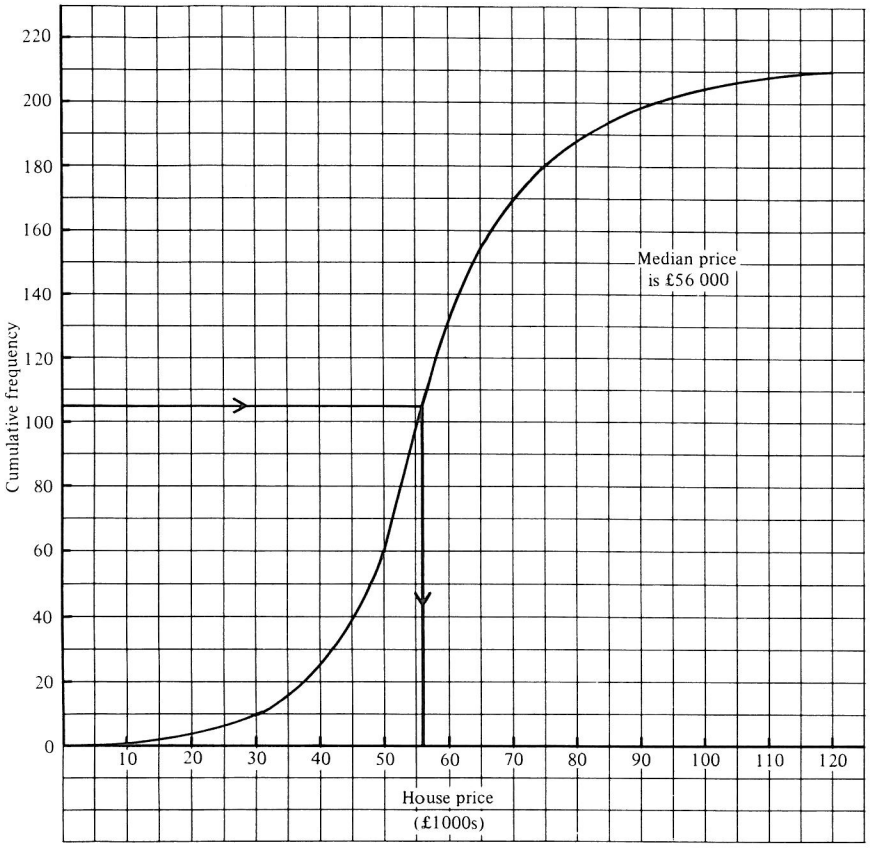
4.


Cumulative frequency
100
90
80
70
60
50
40
30
20
10
67
68
69
70
71
72
73
74
75
76
77
78
Score

Exercise 13e (page 235)

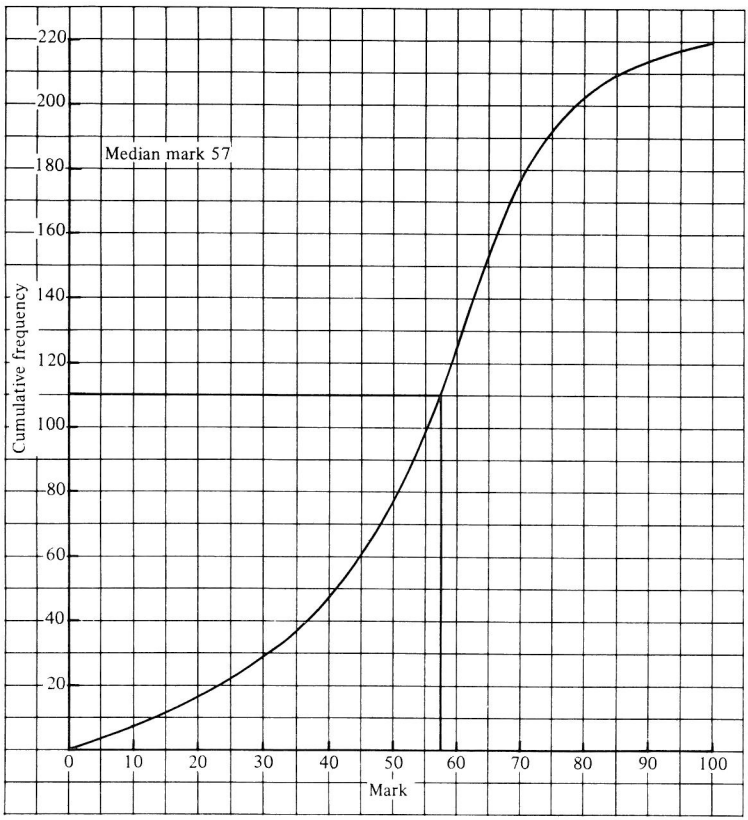
1. 1.7, 48, 43, 71.9

2.



3. a) 80 b) 48

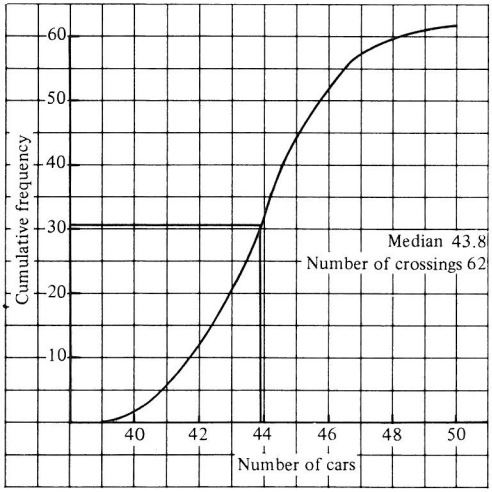
4.


Median mark 57
Cumulative frequency
220
200
180
160
140
120
100
80
60
40
20
0
10
20
30
40
50
60
70
80
90
100
Mark

5.

Number of cars	$\leqslant 40$	$\leqslant 41$	$\leqslant 42$	$\leqslant 43$	$\leqslant 44$	$\leqslant 45$
Number of crossings	2	6	12	22	32	44

Number of cars	$\leqslant 46$	$\leqslant 47$	$\leqslant 48$	$\leqslant 49$	$\leqslant 50$
Number of crossings	52	58	60	61	62



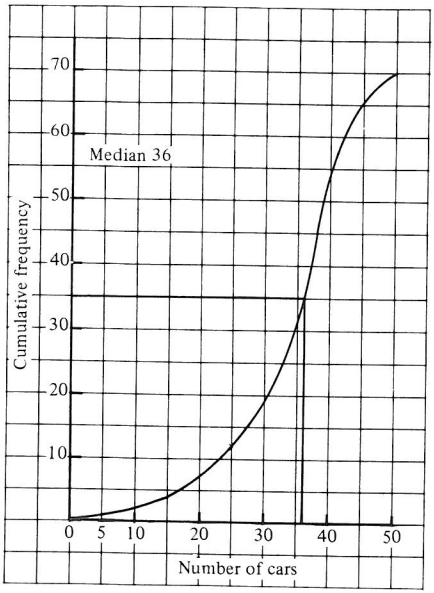
6.

Number of cars	0–5	6–10	11–15	16–20	21–25
Frequency	1	1	2	3	5

Number of cars	26–30	31–35	36–40	41–45	46–50
Frequency	7	13	22	11	5

Number of cars	$\leqslant 5$	$\leqslant 10$	$\leqslant 15$	$\leqslant 20$	$\leqslant 25$
Cumulative frequency	1	2	4	7	12

Number of cars	$\leqslant 30$	$\leqslant 35$	$\leqslant 40$	$\leqslant 45$	$\leqslant 50$
Cumulative frequency	19	32	54	65	70



Exercise 13f
(page 239)

1.

	Upper quartile	Lower quartile	Interquartile range
Question 4	67	42	25
Question 5	45.4	42.5	2.9
Question 6	40	29	11

2. a) 23
b) lower quartile is 16
upper quartile is 34.5
interquartile range is 18.5

3. a) 328 b) 62 c) 74, 52, 22

4. a) 29 b) 34.5, 23

5. a) 37 b) 50, 27, 23 c) 50

6. a)

Score	67	68	69	70	71	72	73	74
Frequency	4	7	9	9	6	3	1	1

b)

Score	$\leqslant 67$	$\leqslant 68$	$\leqslant 69$	$\leqslant 70$	$\leqslant 71$	$\leqslant 72$	$\leqslant 73$	$\leqslant 74$
Cumulative frequency	4	11	20	29	35	38	39	40

c) 20 d) 20 e) 70

7. a)

Score	$\leqslant 10$	$\leqslant 20$	$\leqslant 30$	$\leqslant 40$	$\leqslant 50$	$\leqslant 60$
Cumulative frequency	7	16	27	40	56	74

Score	$\leqslant 70$	$\leqslant 80$	$\leqslant 90$	$\leqslant 100$	$\leqslant 110$	$\leqslant 120$
Cumulative frequency	85	92	96	98	99	100

b) 56 c) 15 d) 46, 61, 28, 33

8. a) 11.7 cm
b) 12.4 cm and 10.8 cm

Exercise 13g (page 243)

1. a) 53 b) 90 c) graph d) 58; 68, 43 e) $\frac{89}{250}$, $\frac{108}{250}$

2. a) 22 b) 14 c) graph
d) median 74, $Q_3 = 80$, $Q_1 = 61$ e) $\frac{3}{5}$

3. a) 20 b) £2.60
c) One quarter had less than £1.50 per week
d) £4.20. A vertical line (CD) at £4.20 on the horizontal axis.
The range in the amount of pocket money received by the middle 50% of the 240 pupils is £2.70.
e) £735 f) £3.06

4. a) 32 b) 38, 25 c) 20 d) 27 e) $\frac{3}{8}$

5. graph
b) i) 3550, ii) 56, iii) 21, iv) 28%
c) i) $\frac{3}{16}$, ii) $\frac{33}{80}$

6. a)

Age (years)	$\leqslant 25$	$\leqslant 29$	$\leqslant 33$	$\leqslant 37$	$\leqslant 45$
Cumulative frequency	3	7	15	27	40

b) 7.3 c) i) $\frac{1}{5}$, ii) $\frac{5}{8}$ d) $\frac{7}{195}$, ii) $\frac{1}{15}$

CHAPTER 14 Matrix Transformations

Squared paper can be used for this chapter.

Exercise 14a (page 247)

1. $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ **3.** $\begin{pmatrix} -7 \\ 5 \end{pmatrix}$ **5.** $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$

2. $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ **4.** $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$ **6.** $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$

7. (5, 2) **9.** (−2, −4) **11.** (−6, 2)
8. (1, −3) **10.** (2, −3) **12.** (2, −6)

Exercise 14b (page 248)

The drawings are clearer if the objects are drawn in one colour and the images in another.

1. $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$ **3.** $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$ **5.** $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$

2. $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$ **4.** $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ **6.** $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$

Exercise 14c (page 250)

1. $\begin{pmatrix} 5 \\ 11 \end{pmatrix}$ **3.** $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ **5.** $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$

2. $\begin{pmatrix} 9 \\ 6 \end{pmatrix}$ **4.** $\begin{pmatrix} -2 \\ -5 \end{pmatrix}$ **6.** $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$

7. A′(1, −1), B′(3, −3)
8. A′(−1, −1), B′(2, 4)
9. A′(10, 3), B′(−5, −2)
10. A′(3, 3), B′(−6, 0)
11. A′(−3, 2), B′(3, −7)
12. A′(7, 4), B′(1, −8)

Exercise 14d (page 253)

There are only four commonly used reflections so some of them are bound to crop up twice. This could encourage the pupils to notice that the same transformation can act on two different objects to produce two different images but the *transformation* is still the same.

1. A′(−2, −1), B′(2, −1), C′(3, −2), D′(−1, −2); reflection in x-axis
2. A′(1, 1), B′(1, 4), C′(2, 4); reflection in line $y = x$
3. A′(−2, −3), B′(−5, −3), C′(−3, 2); reflection in y-axis
4. A′(−1, −4), B′(−3, −3), C′(0, −2); reflection in line $y = -x$
5. A′(1, 1), B′(1, 3), C′(2, 3), D′(2, 1); reflection in line $y = x$
6. A′(0, 2), B′(0, 4), C′(2, 4), D′(2, 2); reflection in line $y = x$
7. A′(−1, −1), B′(−1, −2), C′(−2, −2), D′(−2, −1); reflection in line $y = -x$
8. A′(−1, 0), B′(−4, 0), C′(−4, 2); reflection in y-axis
9. A′(2, −1), B′(3, −1), C′(3, −4), D′(2, −4); reflection in x-axis
10. A′(1, 1), B′(1, 3), C′(3, 4), D′(3, 3); reflection in line $y = x$
11. A′(−2, 4), B′(−4, 5), C′(−3, 2); reflection in y-axis

Exercise 14e (page 257)

1. A′(1, −1), B′(1, −4), C′(3, −4), D′(3, −1); rotation of 90° clockwise about O
2. A′(−1, −1), B′(−4, −1), C′(−4, −2), D′(−1, −2); rotation of 180° about O
3. A′(0, 1), B′(0, 3), C′(−4, 4); rotation of 90° anticlockwise about O
4. A′(1, −1), B′(1, −4), C′(4, −4); rotation of 90° clockwise about O
5. A′(−3, −2), B′(−4, −3), C′(−1, −4); rotation of 180° about O

Exercise 14f (page 258)

1. A′(2, 0), B′(6, 0), C′(6, 6); enlargement centre O, scale factor 2
2. A′(0, 3), B′(−6, 3), C′(−6, 0), O′(0, 0); enlargement centre O, scale factor 3
3. A′(3, 3), B′(3, 6), C′(6, 6), D′(6, 3); enlargement centre O, scale factor $1\frac{1}{2}$
4. A′(10, 5), B′(10, 10), C′(−10, 10); enlargement centre O, scale factor $2\frac{1}{2}$
5. O′(0, 0), A′(0, −2), B′(2, −2), C′(2, 0); enlargement centre O, scale factor -2
6. A′(0, −2), B′(−3, −2), C′(−3, −5), D′(0, −5); enlargement centre O, scale factor -1; rotation of 180° about O

Exercise 14g (page 260)

1. A′(3, 0), B′(9, 0), C′(9, 2), D′(3, 2); stretch parallel to x-axis, scale factor 3
2. A′(1, 0), B′(3, 0), C′(3, 4), D′(1, 4); stretch parallel to y-axis, scale factor 2
3. A′($1\frac{1}{2}$, 1), B′(6, 1), C′(6, 2), D′($1\frac{1}{2}$, 2); stretch parallel to x-axis, scale factor $1\frac{1}{2}$
4. A′(−2, 3), B′(1, 3), C′(1, 6), D′(1, 6); stretch parallel to y-axis, scale factor 3

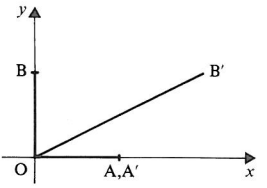
Exercise 14h (page 261)

The transformations given in Numbers 7 and 8 are called shears.

1. A′(−1, −3), B′(1, 3), C′(5, 5), D′(3, −1); (parallelogram)
2. A′(−1, 3), B′(1, −3), C′(−3, −1), D′(−5, 5); (parallelogram)
3. A′(−4, −3), B′(2, −3), C′(2, 6), D′(−4, 6); (rectangle)
4. A′(−8, −4), B′(−2, −1), C′(6, 3), D′(0, 0); (straight line)
5. A′(1, −2), B′(−3, 6), C′(−1, 2), D′(3, −6); (straight line)
6. All points → the origin
7. A′(1, 0), B′(3, 0), C′(6, 2)
8. A′(−2, 0), B′(2, 0), C′(−2, 2), D′(−6, 2)

Exercise 14i (page 262)

In Number 16, we can see that the images of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are given by the columns of the transformation matrix, because $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. A diagram showing the object and the image may therefore be drawn without any calculation. However, not all pupils can recognise the transformation from a diagram showing the position vectors of A, B, A′ and B′ only.

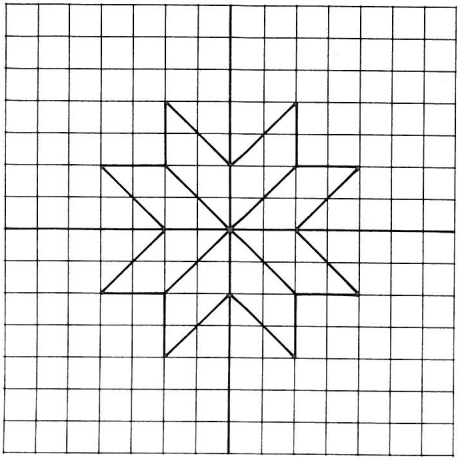


If a transformation has to be identified, the unit square of Number 13 is the best object to choose to use.

1. O′(0, 0), A′(2, 0), B′(2, −1), C′(0, −1); reflection in x-axis
2. O′(0, 0), A′(2, 0), B′($3\frac{1}{2}$, 1), C′($1\frac{1}{2}$, 1)
3. O′(0, 0), A′(0, −2), B′(−1, −2), C′(−1, 0); reflection in line $y = -x$
4. O′(0, 0), A′(−6, 0), B′(−6, 3) C′(0, 3)
5. O′(0, 0), A′(4, 6), B′(1, 8), C′(−3, 2)
6. O′(0, 0), A′(4, 0), B′(4, 1), C′(0, 1); stretch parallel to the x-axis, scale factor 2
7. O′(0, 0), A′(8, 0), B′(8, 4), C′(0, 4); enlargement centre O, scale factor 4
8. O′(0, 0), A′(2, 2), B′(4, 6), C′(2, 4)
9. O′(0, 0), A′(2, 0), B′(2, 3), C′(0, 3); stretch parallel to the y-axis, scale factor 3
10. O′(0, 0), A′(1, 0), B′(1, $\frac{1}{2}$), C′(0, $\frac{1}{2}$); enlargement centre O, scale factor $\frac{1}{2}$
11. O′(0, 0), A′(4 2), B′(6, 5), C′(2, 3)
12. O′(0, 0), A′(2, 4), B′(2, 5), C′(0, 1)

13. The unit square OABC; A(1, 0), B(1, 1), C(0, 1); or the unit triangle OAB

14.



15. The image is the same as the object in each case

16. a) A′(1, 0), B′(0, −1) b) A′(2, 0), B′(0, 2)
c) A′(1, 0), B′(2, 1) d) A′(2, 4), B′(5, −1)
The columns of the matrices give the position vectors of A′ and B′

Exercise 14j (page 265)

1. Rotation of 90° anticlockwise about the origin

2. Enlargement centre O and scale factor $\frac{1}{3}$

3. Reflection in the x-axis

4. Rotation of 45° clockwise about the origin

Exercise 14k (page 266)

1. a) A′(0, 1), B′(0, 3), C′(−3, 3)
b) Rotation of 90° anticlockwise about the origin
c) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ d) The image of ABC
e) Rotation of 90° clockwise about the origin

2. a) A′(3, 3), B′(6, 3), C′(6, 6), D′(3, 6)
b) Enlargement centre O, with scale factor 3
c) $\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$ d) The image is ABCD
e) Enlargement centre O, scale factor $\frac{1}{3}$; Yes

3. a) A′(0, −1), B′(1, −3), C′(2, −3), D′(1, −1)
b) Rotation of 90° clockwise about the origin
c) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ d) The image is ABCD
e) Rotation of 90° anticlockwise about the origin. Yes

4. a) A′(3, 1), B′(9, 3), C′(7, 3)
b) $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ c) The image is ABC

5. a) A′(1, 3), B′(3, 9), C′(5, 13), D′(3, 7)

b) $\begin{pmatrix} -2 & 1 \\ 3 & -1 \end{pmatrix}$ c) A′B′C′D′ → ABCD

6. a) O′(0, 0), A′(2, 1), B′(10, 5), C′(8, 4); Image is a straight line
b) No inverse c) Transformation has no inverse either

Exercise 14l (page 268)

The results of this exercise may already have been noticed during the previous work so this exercise may not be necessary.

1. O′(0, 0), A′(2, 0), B′(6, 2), C′(4, 2); O and A
2. O′(0, 0), A′(4, 0), B′(4, 4), C′(0, 4); O
3. O′(0, 0), A′(2, 0), B′(2, −2), C′(0, −2); O and A
4. O′(0, 0), A′(4, 2), B′(6, 6), C′(2, 4); O
5. The origin; yes

Exercise 14m (page 269)

1. Translation defined by the vector $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$
2. Translation defined by the vector $\begin{pmatrix} -5 \\ 1 \end{pmatrix}$
3. Translation defined by the vector $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$
4. Translation defined by the vector $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$
5. O′(3, 1), A′(4, 1), B′(4, 3), C′(3, 3)
6. A′(−1, −2), B′(−1, 0), C′(−2, 0)
7. A′(1, −1), B′(2, −1), C′(2, 1)
8. A′(0, 1), B′(1, 1), C′(1, 3), D′(0, 3)
9. Translations defined by the vectors:

1. $\begin{pmatrix} -5 \\ -1 \end{pmatrix}$ 2. $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$ 3. $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ 4. $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

5. $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$ 6. $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$ 7. $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$ 8. $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

CHAPTER 15 Quadratic Equations

Exercise 15a (page 272)

Much of this exercise can be considered orally.

1. $(x + 3)^2$
2. $(a + 2)^2$
3. $(p - 5)^2$
4. $(s - 6)^2$
5. $(x - \frac{5}{2})^2$
6. $(b + \frac{3}{2})^2$
7. $(x + \frac{9}{2})^2$
8. $(x - \frac{1}{2})^2$
9. $(x - \frac{1}{4})^2$
10. $(x + 4)^2$
11. $(x + \frac{1}{2})^2$
12. $(x + \frac{1}{3})^2$
13. $(p + 9)^2$
14. $(a - \frac{2}{5})^2$
15. $(t - \frac{3}{4})^2$

16. $(x + b)^2$
17. $(x - c)^2$
18. $(x + \frac{b}{2a})^2$
19. $(3x + 1)^2$
20. $(2x - 3)^2$
21. $(10x - 3)^2$
22. $(3x - 4)^2$
23. $(2x - 1)^2$
24. $(5x + 2)^2$
25. $(3x - 1)^2$
26. $(2x + \frac{1}{2})^2$
27. $(\frac{3}{2}x + \frac{2}{3})^2$

Exercise 15b (page 274)

Best used as an oral exercise with the addition of home produced examples if necessary.

1. 4
2. 16
3. 36
4. 49
5. $\frac{9}{4}$
6. 100
7. $\frac{49}{4}$
8. $\frac{1}{16}$
9. $\frac{9}{16}$
10. $\frac{1}{4}$
11. h^2
12. $\frac{b^2}{4a^2}$

Exercise 15c (page 275)

This exercise may be omitted on first reading.

1. 4
2. 9
3. 25
4. 9
5. 4
6. 25
7. 4
8. 4
9. $\frac{1}{4}$

Exercise 15d (page 275)

Show that, if $x^2 = 4$, then writing $x = \pm 2$ or $\pm x = \pm 2$ gives the same information.
Most pupils need to be satisfied on this point at some time or another.

1. 2, −4
2. −2, 6
3. −2, 8
4. −16, 4
5. −8, −6
6. −4, 6
7. −9, 5
8. 1, 9
9. 5, 9
10. −8, 0
11. −8, 2
12. 3, 15
13. $-\frac{1}{2}$, $-1\frac{1}{2}$
14. $\frac{1}{2}$, $3\frac{1}{2}$
15. −2, 3
16. $-\frac{3}{2}$, $\frac{5}{2}$
17. $-\frac{7}{3}$, 1
18. −1, $\frac{7}{5}$
19. 1, $\frac{5}{3}$
20. $-\frac{12}{7}$, $\frac{8}{7}$
21. $-\frac{7}{2}$, $\frac{5}{2}$
22. −1, $\frac{11}{3}$
23. $-\frac{7}{5}$, $\frac{3}{5}$
24. $\frac{1}{2}$, 1
25. $\frac{3}{9}$, $\frac{7}{9}$
26. $-1\frac{2}{5}$, $\frac{1}{5}$
27. $-\frac{4}{7}$, 2

Exercise 15e (page 277)

Better pupils should profit by being shown how to solve a quadratic equation by completing the square even if it is subsequently discarded in favour of the formula.

1. 1, −5
2. −1, 7
3. −11, 1
4. −7.61, −0.39
5. 0.27, 3.73
6. −8.36, 0.36
7. −1.61, 5.61
8. −8.53, −0.47
9. 0.81, 6.19
10. −1.56, 2.56
11. −9.32, 0.32
12. −0.54, 7.46
13. −4.10, 1.10
14. −0.35, 2.35
15. −2.32, 0.32
16. −0.85, 2.35
17. −4.58, 0.58
18. −0.18, 1.85
19. −0.52, 1.52
20. −1.29, −0.31
21. −0.36, 2.11
22. −0.17, 1
23. −1.41, 0.41
24. −0.21, 3.21

Exercise 15f (page 280)

Pupils should be encouraged to check that the sum of the roots is equal to $-\frac{b}{a}$

1. −5.45, −0.55
2. −6.37, −0.63
3. −3.62, −1.38
4. −7.27, 0.27
5. −4.65, 0.65
6. −7.37, −1.63
7. −5.73, −2.27
8. −11.32, 1.32
9. −6.87, 0.87
10. −9.11, 0.11
11. −4.19, 1.19
12. −5.32, 1.32
13. 3.41, 0.59
14. 0.46, 6.54
15. 1.27, 4.73
16. −0.65, 4.65
17. −0.85, 5.85
18. 0.44, 4.56
19. 0.38, 2.62
20. −0.41, 7.41
21. −0.22, 9.22
22. −1.61, 5.61
23. −7.27, 0.27
24. −7.32, −0.68
25. −3.19, −0.31
26. −2.78, −0.72
27. −1.77, −0.57
28. −1.59, −0.16
29. −1.54, −0.26
30. 0.72, 2.78
31. 0.16, 1.59
32. 0.26, 1.54
33. −2.14, 0.47
34. −3.11, 0.11

Exercise 15g (page 282)

1. −1.08, −5.08
2. −0.32, 2.32
3. −2.14, 0.47
4. −0.68, 0.88
5. 0.36, 1.39
6. −0.16, 4.16
7. −0.28, 1.78
8. −1.55, 0.80
9. 0.24, 2.76
10. −0.36, 1.86
11. −0.77, 3.27
12. −1.55, 0.22
13. −0.21, 1.21
14. −2.59, 0.26
15. 0.28, 0.72
16. −0.30, 0.42

Exercise 15h (page 283)

Some of the equations included in this exercise may be solved by factorising. This is deliberate since factorisation should always be attempted before resorting to the formula. Some examination boards employ a code. When they ask for the roots of a quadratic equation to be given 'correct to two decimal places', they mean 'use the formula'!

1. −2, 0.5
2. −1.58, −0.42
3. −0.67, −0.5
4. −2.19, 0.69
5. 0.28, 2.39
6. −0.33, 3
7. −0.69, 2.19
8. −1.50, 0.25
9. −1.40, 0.24
10. −0.43, 1.18
11. −1.35, 0.21
12. −0.24, 0.84
13. 0.33, 2.00
14. −0.70, −0.39
15. −0.75, 0.20
16. 1.67, 3
17. −1.29, −0.31
18. −0.39, 3.89
19. −5, 0.5
20. −0.30, 1.13

Exercise 15i (page 284)

This exercise includes demanding questions. The better pupils will enjoy tackling them, and their solution will improve the pupils' manipulative skills.

1. 0.18, 10.82
2. −1.19, 4.19
3. 0.30, 6.70
4. −4.30, −0.70
5. −4.55, 2.80
6. −0.84, 0.59
7. −4.27, 3.27
8. −3.27, 0.77

Exercise 15j (page 285)

Plenty of time spent in classroom discussion should result in more acceptable solutions.

1. 6, 7
2. 2, 4
3. 5, 6
4. 5 cm, 9 cm
5. 5 cm, 8 cm
6. 5, 8
7. 5 cm, 12 cm, 13 cm
8. 2 cm, 8 cm
9. 6 cm, 8 cm, 10 cm
10. 6 cm, 8 cm
11. 7, 12
12. 8, 11
13. 4 cm, 9 cm
14. a) 12 cm
    b) 7 cm
15. 13 cm by 6 cm
16. 24 cm, 5 cm

Exercise 15k (page 287)

1. 1.13, 8.87
2. 1.17, 7.83
3. 0.05 or 19.95
4. 6.22, 3.22
5. 8.46 cm, 6.46 cm, 5.46 cm
6. Parallel sides are 4 cm and 10 cm
   Distance between them is 6 cm
7. 10 cm × 5 cm × 3 cm
8. 38 years

Exercise 15l (page 288)

An interesting exercise for the better pupil.

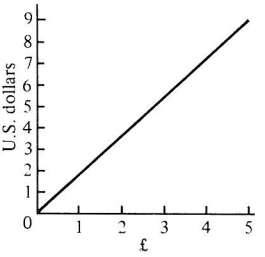
1. 60 mph
2. 30 p
3. £6, £9
4. £1 ≡ 10 F
5. Rectangle 9 cm × 6 cm, square of side 3 cm
   or Rectangle $\frac{27}{7}$ cm × $\frac{18}{7}$ cm, square of side $\frac{51}{7}$ cm
6. 42

CHAPTER 16 Using Money

Revise basic percentage work before working this chapter. Much of the work in this chapter is optional. However it does provide practice in arithmetic and some of the topics covered are useful general knowledge.

Exercise 16a (page 291)

1. a) £20 b) £20.10
2. a) £2 b) £2.01
3. a) £250 b) £251.26
4. a) 50 p b) 50 p
5. a) £45 b) £45.23
6. a) £25 b) £27.78
7. a) 50 p b) 56 p
8. a) 20 p b) 22 p
9. a) £3 b) £3.33
10. a) £19 b) £21.17
11. a) £1.50 b) £1.38
12. a) £5 b) £4.59
13. a) £$\frac{1}{8}$, 12$\frac{1}{2}$ p b) 11 p
14. a) £6.25 b) £5.73
15. a) 50 p b) 41 p
16. a) £2 b) £2.08
17. a) £4 b) £4
18. a) £8 b) £8.33
19. a) £20 b) £20.83
20. a) £1$\frac{1}{3}$ b) £1.33
21. a) £3 b) £3.08
22. 8.29 Ff
23. 5.53 Ff
24. 219 L
25. 8.26 pta
26. 121 L
27. 29.02 Ff
28. 35.93 Ff
29. 5477 L
30. 372 pta
31. 3391 L
32. 18 404 L
33. 314 pta
34. 8.88 DM
35. 7.40 DM
36. 4.14 DM
37. £6.08
38. £2.70
39. 84 p
40.



a) \$4.50 b) \$8.55 c) £2.33

Exercise 16b (page 294)

Useful table reading practice.

1. £1 = 1.993 C$
2. 1 DM = 749.0 L
3. 100 B Fr = 384.9 yen: 1 B Fr = 3.849 yen
4. 1 N Fl = 0.541 $
5. 1 $ = 129.8 yen
6. 1 S Fr = £0.389
7. 1 $ = £0.564
8. 100 B Fr = 2.966 $: 1 B Fr = 0.029 66 $
9. 1 DM = 0.609 $
10. 1 N Fl = £0.305

Exercise 16c (page 295)

Point out that looking at 'Bank buys and bank sells' columns, it appears at first sight that you get better value on cashing in foreign currency, but don't be misled!

1. 283.5 DM
2. £82.37
3. £469.48
4. £272.48
5. 88 500 pta
6. £78.61
7. £417.36
8. £8.67
9. £59.58
10. £226

Exercise 16d (page 296)

Point out that the interest payable on an investment is only one factor that influences choice of investment. Ease of access to capital and withdrawal facilities also matter.

1. a) £105.06 b) £104.06
2. a) £109.20 b) £109.20
3. a) £214.25 b) 7.12%
4. a) £110.25 b) 10.25% c) £121.55
5. £119.25; not as good as the Savings Certificates
6. £200; £1169.86
7. The local authority bond pays £90 a year; the savings account gives £87.75 interest a year; the bond gives the greater return

Exercise 16e (page 298)

1. 9.86% **2.** 12.3% **3.** 6.25% **4.** 8.57% **5.** 20% **6.** 27.3% **7.** 7.5%

8. The net rate on the savings account is 5.6%, so a tax payer gets 0.4% more from the building society
9. 10% gross is equivalent to 6.7% net of standard rate tax.
   a) net account, by £26 a year
   b) gross account, by £40 a year

Exercise 16f (page 301)

Nos. 1–4 can be worked before discussing multiplying factors. If more simple problems are required before embarking on multiplying factors, Exercise 5g in 3A can be used.

1. £102.50
2. £561.80
3. £779.14
4. £496.50
5. 2.476
6. 1.851
7. 0.2725
8. 0.6302
9. 1.685
10. 16.37
11. 0.4344
12. 0.1880
13. £3173.75

14. a) £11 655.26 b) £21 306.27
15. £11 390.81
16. a) £4962.81 b) £4603.02
17. a) £3711.71 b) £4235.94
18. a) £72 600 b) £96 600
19. a) £68 000 b) £79 300

20. £100.57
21. a) £1100 b) £900
22. £14.40
23. 2 400 000

Exercise 16g (page 303)

1. £18.51
2. £37.40
3. 3582
4. £171.80
5. 8630
6. £1019
7. £803
8. £1651
9. £609.90
10. £823.60
11. £200
12. £500
13. £900
14. 4000
15. £131.22
16. 4
17. 8
18. 9
19. 3
20. 7
21. 14%
22. 3

Exercise 16h (page 306)

It is worth mentioning charge cards as well as credit cards. For example American Express is a charge card: on such cards the monthly balance has to be paid in full; there is no extended credit (the card company usually charges a fixed annual amount for the card so they get their money from that rather than from interest).

1. £575
2. a) £184 500 b) £118 800
3. a) £775.20 b) £232 560
4. £291.67
5. Yes: monthly repayments of £82.50 needed to cover interest
6. £626
7. £128
8. a) £5 b) £300
9. a) £10 129.68 b) £9500.10
10. Rental costs £4320 and buying and paying for repairs costs £4360 so buying would cost £40 more
11. £18.11
12. No: he only had £26.80 credit available

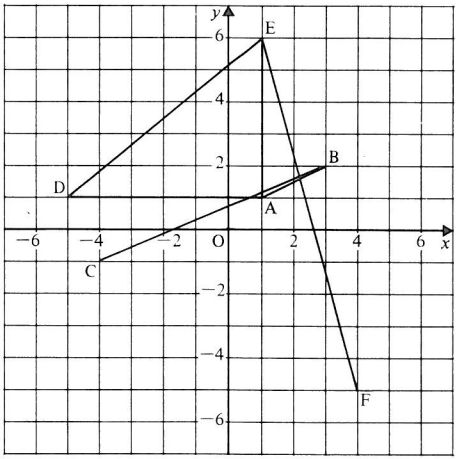
CHAPTER 17 Gradients and Areas

If further practice in finding areas and gradients is required, use graphs drawn for chapter 5 or any other previously drawn graphs.
(The graphs provided for these answers are to give an idea of the shape required. They are drawn to smaller scales than those asked for in the questions.)

Exercise 17a (page 310)

Drawing a curve on the board and moving a ruler along the curve in the direction of the tangent, helps show how the gradient at any point is the gradient of the tangent.

1.

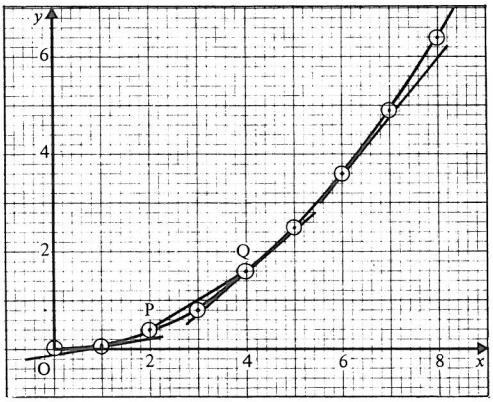


a) $\frac{1}{2}$
b) $\frac{3}{7}$
c) $\frac{5}{6}$
d) $-\frac{11}{3}$
e) 0
f) not possible

2. a) −3 b) 0 c) $\frac{1}{2}$ d) 2

3.

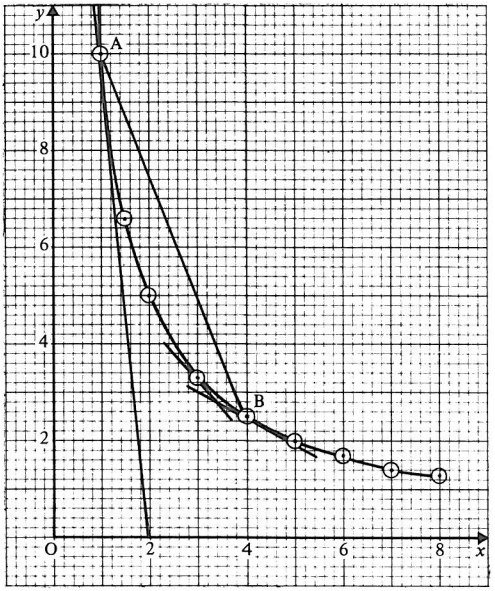
x	0	1	2	3	4	5	6	7	8
y	0	0.1	0.4	0.9	1.6	2.5	3.6	4.9	6.4



a) 0.6
c) $\frac{1}{5}$, $\frac{4}{5}$, $\frac{6}{5}$

4.

x	1	1.5	2	3	4	5	6	7	8
y	10	6.6	5	3.3	2.5	2	1.7	1.4	1.3

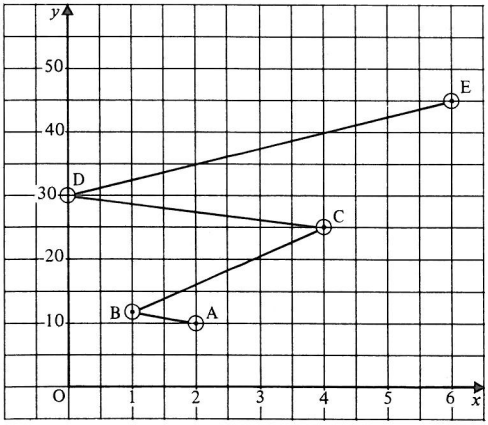


a) −2.5
b) −10, −0.6
c) −1.1

Exercise 17b
(page 312)

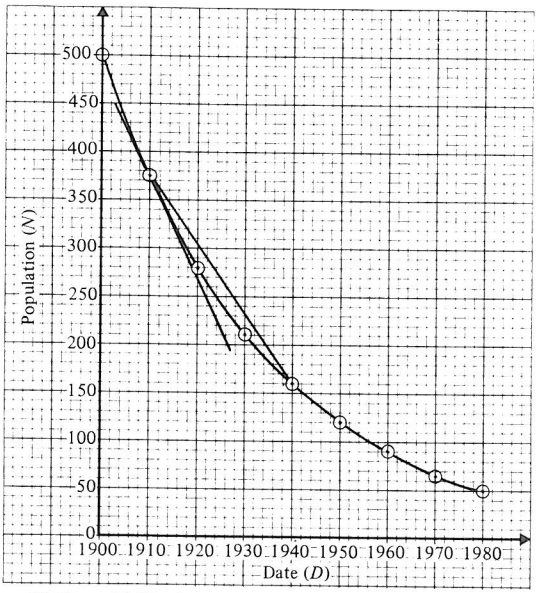
1. a) i) 29 ii) 33
b) 33 or 44 (must be a whole number)
c) $\frac{21}{4}$; the number of ripe strawberries is increasing by $5\frac{1}{4}$ a day on average
d) −6; the number of ripe strawberries is falling at 6 per day

2.



$-2, \frac{13}{3}, -\frac{5}{4}, \frac{5}{2}$

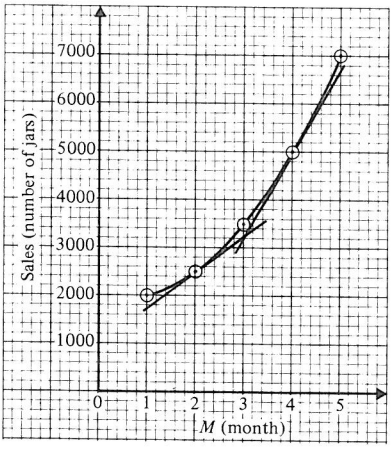
3.



a) –7.2; from 1910 to 1940; the population decreased by an average of 7.2 people per year

b) –11; in 1910; the population was decreasing by 11 people a year

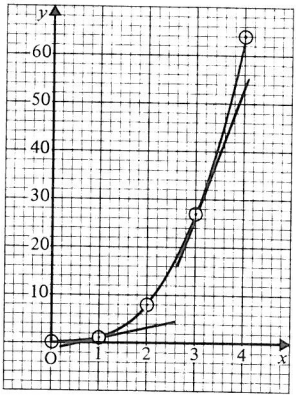
4.



a) 800; in month 2, sales increased by 800 jars a month

b) 1800; in month 4, sales increased by 1800 jars a month

5.

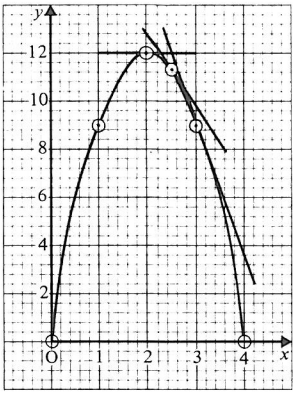


a) 3

b) 27

6.

x	0	1	2	2.5	3	4
y	0	9	12	11.25	9	0



a) 0
b) −3
c) −6

Exercise 17c (page 316)

1. a) 6.25 cm^2 b) 6.25 m^2 (25 squares)
2. a) 10 cm^2 b) 160 m^2 (40 squares)
3. a) 16.75 cm^2 b) 67 km^2 (67 squares)
4. a) 79 cm^2 b) 78.5 cm^2

Exercise 17d (page 318)

Remind pupils of the significance of m and c in the operation $y = mx + c$ and insist on sketches, not accurate plots.

1. 12 sq units
2. 12.5 sq units
3. 36 sq units
4. 9 sq units
5. 78 sq units
6. 112 sq units
7. 60 sq units
8. 33.75 sq units

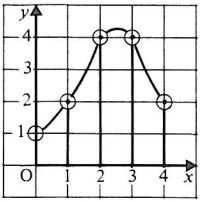
Exercise 17e (page 320)

Sketch graphs are all that is needed for this exercise. This method can be formalised into the trapezium rule:

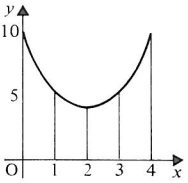
area $\approx \frac{1}{2}d\{$sum of 1st and last ordinate + twice sum of the other ordinates$\}$

where d is the width of each strip.

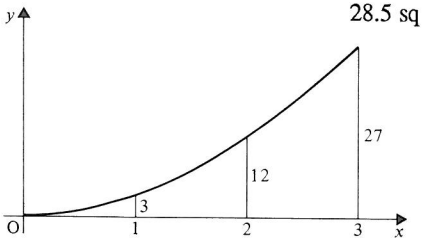
1. a) 26 sq units b) 27 sq units
   The answer to b) is probably nearer the true value
2. 14 sq units
   Using 2 strips gives 12.5 sq units: less accurate because the second strip leaves out a larger area than the extra included by the first trapezium

3. 

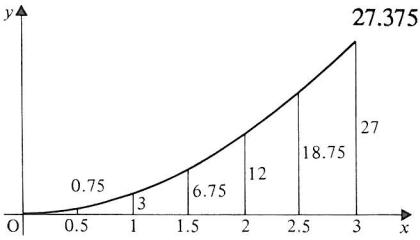
11.5 sq units

4. 

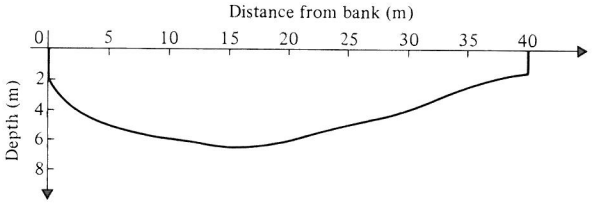
26 sq units and this is greater than the true value

5. a) 

28.5 sq units

b) 

27.375 sq units

6. a) 

b) 183.75 m^2 c) 460 litres d) 1 654 000 litres

CHAPTER 18 Travel Graphs

Exercise 18a (page 323)

Revises units and change of units. A useful memory aid for the relationship between distance, speed and time is 

1. a) 0.55 km/min b) 33 000 m/h
2. a) 0.1 km/s b) 6000 m/min
3. a) $\frac{2}{3}$ miles/min b) $\frac{1}{90}$ miles/s
4. a) 0.025 km/s b) 25 m/s
5. a) 360 000 m/h b) 360 km/h

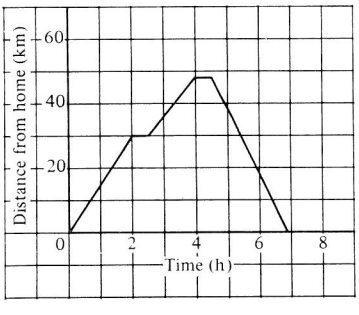
6. 2500 m
7. 0.375 m/s
8. 30 m.p.h.
9. 30 minutes
10. 0.96 km
11. $1\frac{2}{3}$ miles
12. 50 m/s
13. 0.4 m/s
14. 3.6 km
15. 40 seconds

Exercise 18b (page 326)

If it is thought desirable, the distinction between distance and displacement can be made now. The word displacement is not used in this chapter. A common mistake is to use place names on the vertical axis: this is confusing because pupils may tend to think of those points on the vertical axis as 'places' to get to and may even try to make the graph come back to the vertical axis.

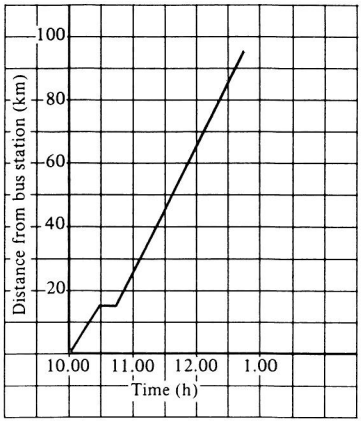
The graphs given in the following answers are drawn to scales smaller than asked for in the questions. They are provided to give an idea of the shape required.

1.



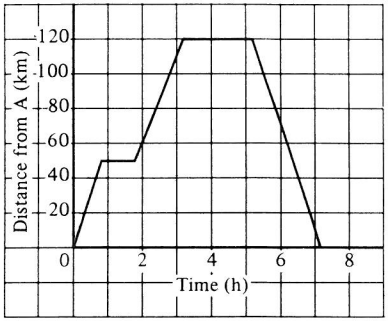
6.9 hours
(6 hours 54 mins)

2.



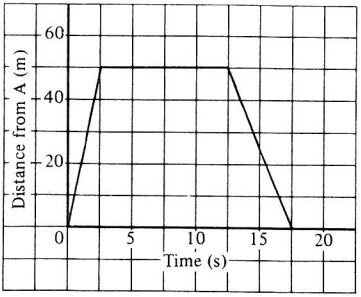
95 km

3.



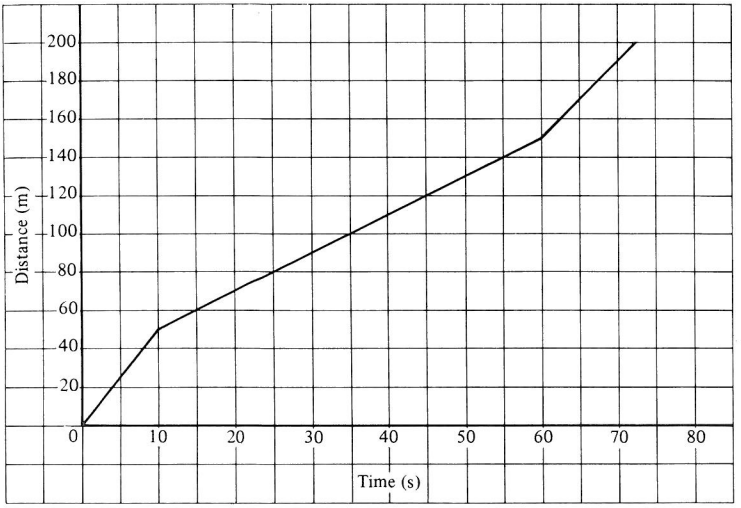
a) 62.5 km/h
b) 2 hours

4.



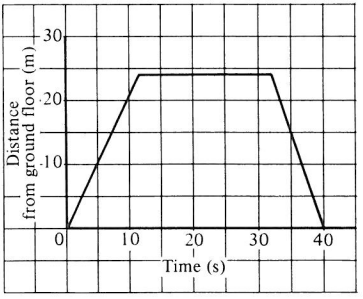
17.5 seconds

5.



72.5 seconds

6.



a) 24 m
b) 3 m/s

Exercise 18c (page 327)

1. a) 12 km/h b) 13.9 km/h (to 3 s.f.)

2. a) 30.4 km/h (to 3 s.f.) b) 35.2 km/h (to 3 s.f.)

3. a) $33\frac{1}{3}$ km/h b) $33\frac{1}{3}$ km/h

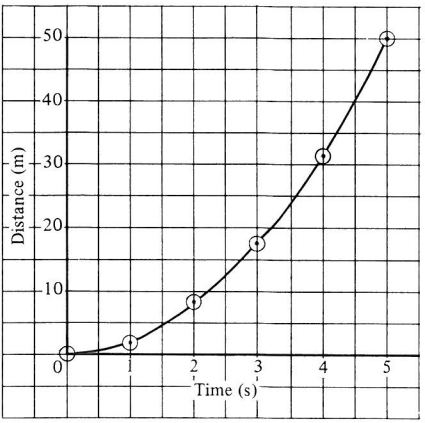
4. a) 5.71 m/s (to 3 s.f.) b) 5.71 m/s (to 3 s.f.)

5. a) 2.83 m/s (to 3 s.f.) b) 2.76 m/s (to 3 s.f.)

6. a) 1.2 m/s b) 1.2 m/s

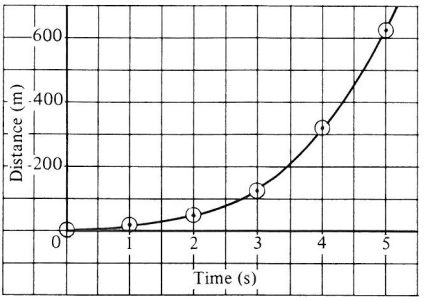
Exercise 18d
(page 329)

1.



a) 12.5 m
b) 6 m/s
c) 10 m/s

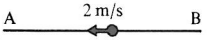
2.

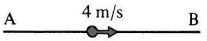


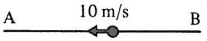
a) 456 m
b) 80 m/s
c) 185 m/s

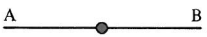
Exercise 18e
(page 330)

1. a) speed b) velocity c) velocity d) speed
e) speed f) velocity g) velocity

2. a) 

b) 

c) 

d) 

3. a) 10 m/s b) 10 m/s c) 7.5 m/s
d) –7.5 m/s e) 6 m/s

4. The ball moves with velocity 0.8 m/s for 5 seconds then with velocity – 0.4 m/s (i.e. in the opposite direction with speed 0.4 m/s) for 5 seconds and then stops.

5. (b), (d), (e)

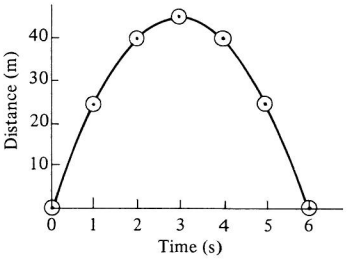
Exercise 18f (page 335)

Point out that drawing a tangent by eye is neither easy or accurate so that answers obtained are very approximate. However it is the only method available at this stage.

1. a) 8 m/s b) 8 m/s c) 16 m/s

2. a) 140 m/s b) 95 m/s c) 60 m/s d) 93.75 m/s (94 m/s)

3.



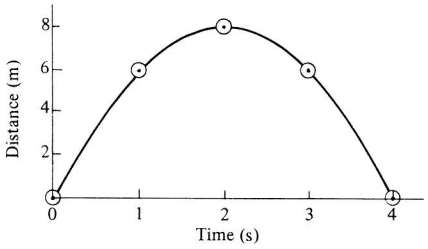
a) 6 sec after leaving it
b) 15 m/s
c) 17.5 m/s
d) 20 m/s
e) −10 m/s
f) −20 m/s

4. a) 15 m/s b) 20 m/s c) 5 m/s d) 31 m e) −15m/s

5. a)

t	0	1	2	3	4
$8t$	0	8	16	24	32
$-2t^2$	-0	-2	-8	-18	-32
d	0	6	8	6	0

b)



c) 0, −4 m/s d) 8 m

Exercise 18g (page 337)

1. 30 m/s, 60 seconds

2. a) 4 m/s b) 60 m/s c) 120 m/s

3. 15 seconds

4. 2 m/s^2

5. a) 90 km/h b) 300 km/h c) 22.5 km/h

6. 14 km/h/s ≡ 14 km/minute^2 (accept any units, e.g. $3\frac{8}{9}$ m/s^2)

7. $2\frac{7}{9}$ m/s^2

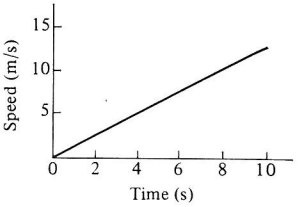
Exercise 18h
(page 339)

1. a) 2.5 m/s^2 b) the acceleration becomes less
c) 2 seconds d) 2 seconds
e) 7.5 m/s^2 f) 12 seconds

2. a) 10 km/min^2 b) 15 km/min
c) 27.5 km/min d) 2.5 km/min^2

3. a) 5 km/h/min $\equiv \frac{1}{12}$km/min$^2 \equiv$ 300 km/h^2
b) 15 km/h c) 3 minutes d) zero
e) 5 km/h/min $= \frac{1}{12}$km/min$^2 \equiv$ 300 km/h^2

4.



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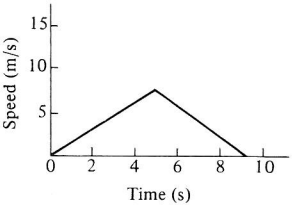

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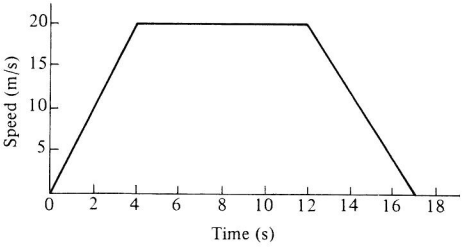

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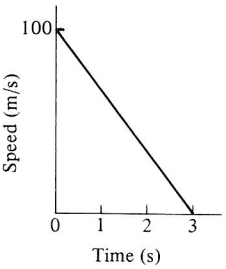

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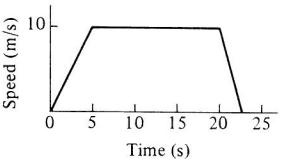

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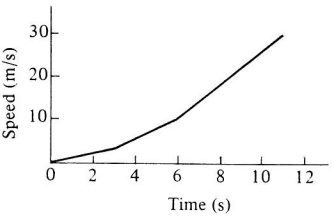

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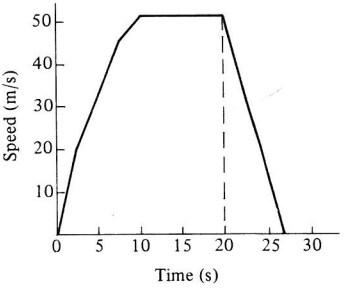

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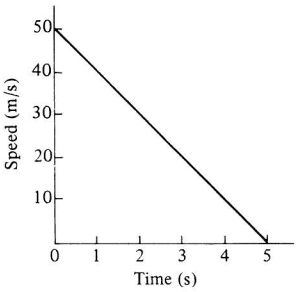

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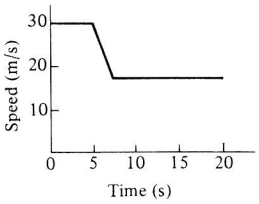

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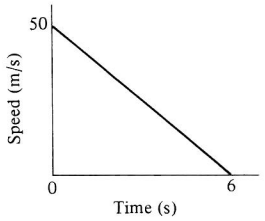


Exercise 18i (page 342)

1. a) 15 m/s^2 b) 7.5 m/s^2 c) 1 sec d) 15 m e) 67.5 m

2. a) 115 m b) 35 km c) $\frac{1}{6}$ km

3. a) 800 m/s^2 b) 3 m
c) 1200 m/s^2 d) 4 m
e) 4000 m/s^2 f) 0.3125 m

4. a) 0.185 m/s^2 b) 1000 m

5. a) 0.017 m/s^2 b) 5 m/s c) 0.025 m/s^2 d) 300 m e) 420 m

6. a) 0.667 m/s^2 b) 1 m/s^2 c) 75 m d) 50 m e) 475 m

An alternative way of saying that the gradient of the tangent gives acceleration is to point out that the gradient of the distance–time curve gives the velocity, so it is reasonable that the gradient of the velocity–time curve gives the acceleration.

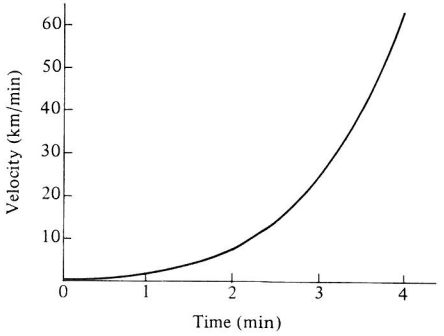
Exercise 18j (page 348)

1. a) 5 m/s b) decelerating c) 38.5 m

2. a) 98 m (using 5 strips)
b) i) T ii) T iii) F iv) T

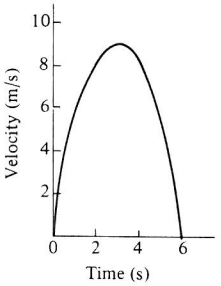
3.

t	0	1	2	3	4
v	0	1	8	27	64



a) 12 km/min^2
b) 16 km/min^2
c) after 2.7 minutes
d) 22.5 km (using 3 strips)
e) 45.5 km (using 1 strip)

4.



a) 9 m/s, 3 seconds after release
b) 7 m/s
c) 4 m/s^2
d) zero
e) 35 m (using 6 strips)

Exercise 18k (page 351)

1. B	**3.** D	**5.** D	**7.** D	**9.** A
2. B	**4.** B	**6.** C	**8.** A	**10.** A

CHAPTER 19 Variation

Much use can be made of real life situations in this chapter.

Exercise 19a (page 354)

1. $y = 3x$

2. $q = p^2$

3. $V = x^3$

4. $r = \sqrt{A}$

5. $xy = 24$ or $y = \frac{24}{x}$

6. $s = \frac{1}{10}r$

7. $y = 4x^2$

8. $pq = -36$ or $q = -\frac{36}{p}$

9. $A = \frac{1}{2}L^2$

10. $A = \frac{1}{3}b^2$

11.

x	2	3	4	5
y	8	27	64	125

Exercise 19b (page 357)

1.

x	2	4	7	8	9.5
y	20	40	70	80	95

$y = 10x$

2.

r	1	3	5	6	8
C	6	18	30	36	48

$C = 6r$

3.

Number of units of electricity used (n)	100	120	142	260	312	460
Total cost in pence (C)	600	720	852	1560	1872	2760

$C = 6n$ The cost of one unit of electricity

4.

Number of oranges bought (X)	2	4	7	9	11	15
Total cost in pence (Y)	20	40	70	90	110	150

$Y = 10X$ The cost of one orange

5. a) 9 b) 16

6. a) $\frac{3}{2}$ b) 20

7. a) 21 b) 7

8. a) 6 b) 3

9. a) 21 b) 40

10. a) 24 b) 15

11. a) 15 b) 8

Exercise 19c (page 359)

1.

x	0	2	3	4	5	8
y	0	12	27	48	75	192

$y = 3x^2$

2.

t	2	4	5	6	10
s	20	80	125	180	500

$s = 5t^2$

3.

x	−3	−1	0	2	4	7
y	36	4	0	16	48	196

$y = 4x^2$

4. a) 32 b) $\pm\frac{1}{2}$ **5.** a) $\frac{3}{4}$ b) $\pm\frac{1}{3}$ **6.** a) 108 b) ± 8

7.

H	2	4	6	8	10
V	2	16	54	128	250

$V = \frac{1}{4}H^3$

8.

x	3	6	9	12	15
y	9	72	243	576	1125

$y = \frac{1}{3}x^3$

9. a) 24 b) 6
10. a) 216 b) 1
11. a) 108 b) 2

12.

R	0	1	4	9	25
V	0	4	8	12	20

$V = 4\sqrt{R}$

13. a) 2 b) 900

14. No;

$\sqrt{x}$	1	2	3	4	5
y	1	2	3	4	5

yes; $y = \sqrt{x}$

15. No; $y = \frac{1}{2}x^3$

Exercise 19d (page 363)

1. $CN = 500$ or $N = \frac{500}{C}$
2. $CN = 720$ or $C = \frac{720}{N}$
3. $PV = 120$ or $V = \frac{120}{P}$
4. $xy = 12$ or $y = \frac{12}{x}$
5. $xy = 72$ or $y = \frac{72}{x}$
6. $xy = 1$ or $y = \frac{1}{x}$

Exercise 19e (page 364)

1. $xy = 36$ or $y = \frac{36}{x}$
2. $y = \frac{36}{x^2}$
3. $q = \frac{60}{\sqrt{p}}$
4. a) 4 b) 20 c) −10
5. a) $\frac{4}{3}$ b) 16
6. a) 10 b) 40
7. a) 6 b) 0
8. a) 25 b) 10
9. a) 4 b) 12
10. a) 56 b) 2
11. a) 2 b) 2 c) 3 d) −1 e) $\frac{1}{2}$ f) 1

Exercise 19f (page 367)

1. a) 1 b) 1 c) $\frac{25}{4}$
2. a) 21 b) 6
3. a) $y = \frac{3}{4}x^3$ b) 6 c) 2
4. a) 14 b) 3
5. a) 8 b) ±3

6.

x	0	1	2	4	8
y	0	0.25	1	4	16

7.

s	0	4	9	16	64
t	0	0.5	0.75	1	2

8. a) 2 b) 3 c) 1 d) –1

Exercise 19g (page 369)

Your science colleagues will be most appreciative of the amount of time and effort that goes into this exercise!

1. a) 2.4 N b) $E = 8.3F$

2. 45 °C

3. a)

I	0.42	0.65	0.89	1.18	1.70	1.88
θ	27.6°	37.8°	43°	56.2°	64.4°	67.3°
$\tan\theta$	0.523	0.801	0.933	1.494	2.087	2.391

c) 1.2

4. a) Fifth one c) $d = 4t$; 4 m s^{-1}

5. 10

Exercise 19h (page 371)

Before leaving this chapter pupils should be well aware that mathematically similar solids are very deceptive in terms of the ratio of their volumes or capacities compared with their heights or any other corresponding linear dimensions. This could lead on to discussion of unit pricing and 'best buys'.

1. a) 4 kg b) 25 cm

2. 64 m

3. a) $\frac{8}{5}$ b) $56\frac{1}{4}$

4. a) 25 cm b) 4.8 N

5. a) £320 b) 4.5 m

6. a) doubles b) $\times 5$

7. a) 0.216 litre b) 20 cm

8. a) 3 b) 120

9. a) 40 mph b) 12 mph

10. a) 2 b) $\frac{1}{2}$ c) 4 d) 8

11. a) +25 % b) –20 % c) $56\frac{1}{4}$ %

CHAPTER 20 General Revision Exercises

The exercises in this chapter comprise a mixed selection of examination type questions. They cover work from earlier books as well as topics from this book.

All answers given to 3 s.f. unless instructed otherwise.

Exercise 20a (page 373)

1. a) £9.74 b) $4\frac{3}{8}$

2. a) $2x(x-3)$ b) $(x-1)(x-3)$ c) $(x-5)(x+2)$

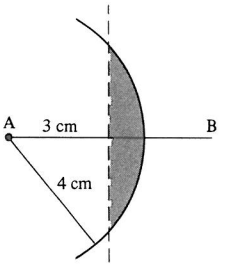
3. a) $\frac{6}{7}$ b) 3, 2 c) $x=3,\ y=-1$

4. a) $8-5x$ b) $\frac{4}{9}$ c) $\frac{x+2}{6}$

5. £35431.22

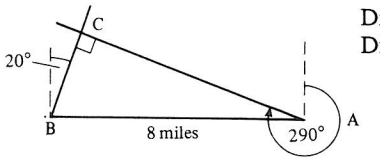
6. $\frac{4}{9}$, $\frac{4}{9}$

7.



8. $y=\frac{144}{x}$ a) 36 b) 16

9.



Distance from A : 7.52 miles
Distance from B : 2.74 miles

10. a) $-\frac{1}{2}$ b) $y=3-\frac{1}{2}x$ c) 3 square units

Exercise 20b (page 374)

1. a) 18.30 b) 15.00

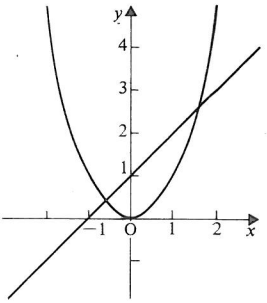
2. a) 75 b) $\frac{1}{5}$ or 1

c) When $t=\frac{1}{5}$ the ball is 1 m from A going up and when $t=1$ the ball is 1 m from A going down

d) $u=\frac{s+5t^2}{t}$

3. a) $15x^3$ b) $\frac{3x+7}{10}$ c) 5

4.



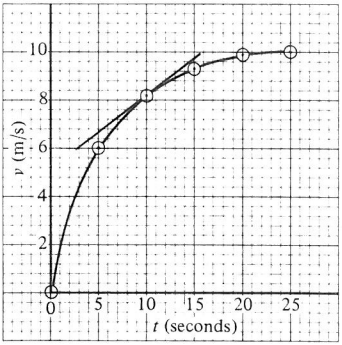
a) $x \approx -0.5$, $x \approx 1.5$
b) $x = -0.62$, $x = 1.62$

5. $x = -6$, $y = 4$

6. a) both 19.1 cm to 3 s.f.
b) 34.9° to 1 d.p.

7. P(3, 1, 2) Q(2, 2, 0) R(1, 2, 2) S(1, 3, 1) T(3, 0, 2) U(3, 0, 0)

8.



a) 7.5 m/s
b) 0.3 m/s/s
c) 50.5 m (using two strips)

9. a) 3.6 cm b) 32 cm^3

10. $x = 70°$, $y = 35°$, $z = 55°$

Exercise 20c (page 376)

1. a) 112 km b) 70 miles

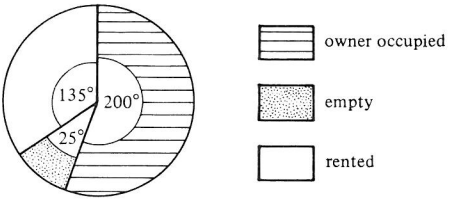
2. a) $x = 3$ b) $x = 3.4$, $y = 1.8$

3. a) $50x + 30y = 320$ or $5x + 3y = 32$
b) $x + y = 8$ c) $x + y < 10$
d) $50x + 30y > 500$ or $5x + 3y > 50$

4. a) 5 b) −3 c) 3

5. 0.42, 3.58

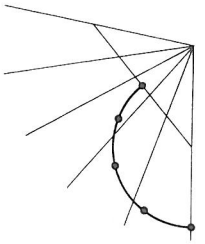
6.



7. a) $\frac{19}{90}$

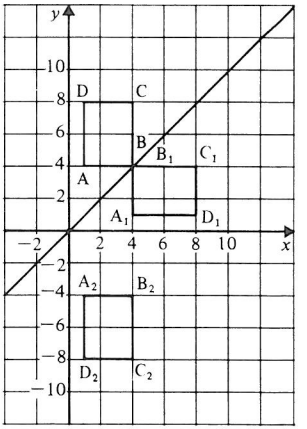
8. $\frac{3}{5}$, $\frac{7}{12}$

9.



$\frac{1}{4}$ scale

10.



$\frac{1}{2}$ scale

c) reflection in x-axis

Exercise 20d (page 378)

1. a) 0.001 53 b) i) 1.53×10^{-3} ii) 0.002

2. a) $\begin{pmatrix} -1 & 16 \\ -5 & 3 \end{pmatrix}$ b) $\begin{pmatrix} 10 & 33 \\ -2 & 24 \end{pmatrix}$ c) 18

3. a) 8 b) 6.37 to 3 s.f. c) 7

4. 41.3 m to 3 s.f.

5. a) $\frac{7t-4}{(t+3)(t-2)}$ b) 1.73, −0.93

6. a) $x = 60$, regular hexagon

b) BC = 17.5 m, $\hat{ACB} = 59.0°$

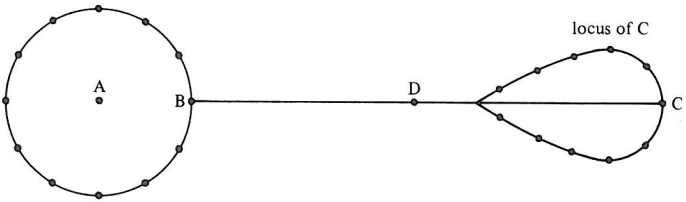
7. △ABC and △YZX (S A S); △PQR and △TUS (A A S)

8. $22 - 4x$ a) $100x$ b) $150x$ c) $5(22 - 4x)$

$100x + 150x + 5(22 - 4x) = 800$, $x = 3$; $3 \times$£1 coins,

9×50 p coins, 10×5 p coins

9.



10.

A	1	4	9	16	25
R	0.2	0.4	0.6	0.8	1

$R = (0.2)\sqrt{A}$, $A = 100$

Exercise 20e (page 380)

1. £ 48.40

2. a) $(x-3)(x+3)$ b) $3x(y-2x)$ c) $2(x-3)(x+1)$

3. a) £10 b) £12.50 c) £ 12

4. a) $\frac{1}{12}$ b) $\frac{5}{9}$ c) $c = \frac{b}{ab-1}$

5. a) 10 000 m^2 i.e. 1 hectare

b) i) 4 cm ii) 6.5 cm iii) 9 cm iv) $8\frac{7}{8}$ cm^2

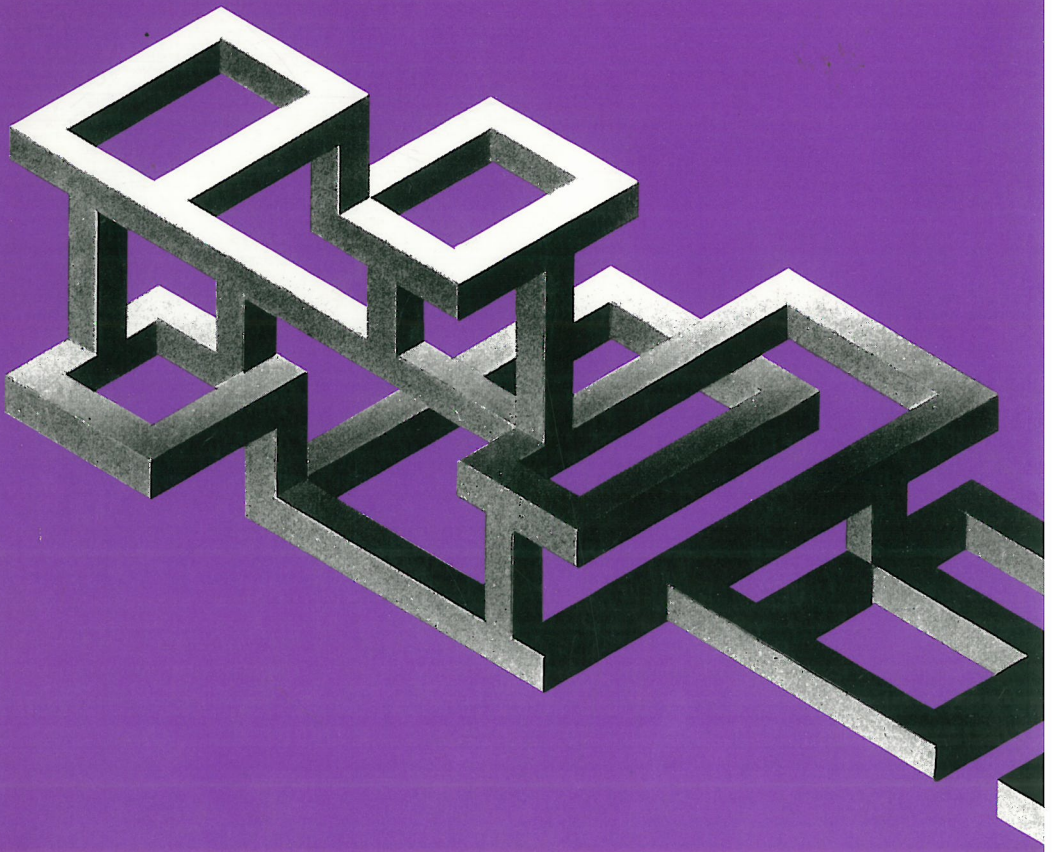
7. $x^2 + (x+2)^2 = 164$; 8, 10

8. a) 14 00 b) 10 00, 1 h 43 min

c) 00 50, 2 h 49 min

9. a) true b) false c) true d) true

10. a) i) 4.77 cm ii) 715 cm^3 b) i) G ii) E iii) 60 cm^3



Stanley Thornes

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